Computation of laser cavity eigenmodes by the use of a 3D finite element approach

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Abstract: A new approach for computing eigenmodes of a laser resonator by the use of finite element analysis (FEA) is presented. The results obtained by this method have been successfully verified by the use of the gaussian mode algorithm.

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1. Introduction

Since the pioneering work of Fox and Li [1] numerical computation of the mode structure of a laser cavity mainly is carried through by the use of a round-trip integral operator being iteratively applied on the field distribution at a certain reference plane. In modern computer codes, the round-trip integral is carried through by a beam propagation method, which is used to compute a series of round-trips starting with a more or less arbitrary initial field distribution. Though widely used, this procedure is not really suited to compute particular eigenmodes of a laser cavity. Even if the procedure converges to the fundamental mode, usually an admixture of higher order modes remains, making it difficult to isolate the exact shape of the fundamental mode. Methods [2, 3] that have been proposed to extract particular eigenmodes by sampling the fluctuating field distributions at the reference plane are time consuming, and due to superimposed numerical fluctuations, usually not very accurate.

In view of these problems, we present a new approach to compute the eigenmodes and eigenvalues of optical resonators that is based on finite element analysis (FEA). Different from the beam propagation codes we are not using an integral equation, but are starting with the underlying differential equation i.e. the scalar wave equation,

$$\left[\Delta + k^2\right]\tilde{E}(x,y,z) = 0 \qquad (1)$$

or Maxwell's equations, if vectorial properties of the electromagnetic field are important. Compared with integral methods, which can use data only at one single reference plane, use of FEA has the advantage that simultaneously the whole resonator domain is involved into the numerical procedure. FEA however faces the difficulty that laser cavities usually are long compared with the wavelength. Therefore, to resolve the oscillations of the propagating wave a huge number of discretization grid points is necessary. To circumvent this problem we separate out the small scale oscillations of the electrical field by the use of a factorization as shown in the next section.

2. Derivation of a solvable eigenvalue problem for the laser modes

A laser cavity is a device in which an electromagnetic wave propagates in a periodically guiding structure. "Guiding structure" means that distributions of a complex valued refractive index and optical elements like lenses and mirrors are grouped along a main axis of propagation in a way that only a small part of the propagating electromagnetic energy is leaking out the sides of the cavity to infinity. "Periodically" means that a substructure of finite length $\tilde{L}$ along the propagation direction exists, and that the full periodic structure is obtained by an identical reproduction of the substructure after $z = n\tilde{L}$ (n = 1, 2, …), where the propagation direction is assumed to coincide with the z-axis. Therefore, we use the factorization

$$\tilde{E}(x,y,z) = \exp\left[-i(k_f - \delta)z\right]\tilde{u}(x,y,z) \qquad (2)$$

for the phasor amplitude $\tilde{E}(x,y,z)$ of the electromagnetic field. Here k_f is the propagation constant of the free wave. The quantity δ takes into account that a guided wave generally has a propagation constant, which is smaller than the propagation constant k_f of the free wave, as well known from texts on wave guide or laser theory (see for instance [4], Chapt. 19.3). If the guiding structure is independent of z, that means if $\tilde{L} \to 0$ as in common wave guides, the phase fluctuations of $\tilde{E}(x,y,z)$ can fully be accounted for by the factor $\exp[-i(k_f - \delta)z]$. In laser cavities, this generally is not the case. For instance, in simple two mirror resonators, the lowest-order gaussian mode shows additional phase fluctuations given by the Guoy phase shift (see [4], Chapt. 19.3)

$$\psi(z) = \arctan \frac{z}{z_R} \tag{3}$$

where z_R is the Rayleigh range. Equation (3) shows that, in this case, the maximum variation of $\psi(z)$ along the resonator axis from the left to the right mirror is confined to $-\pi <= \psi(z) <= \pi$. For higher-order gaussian modes $\psi(z)$ is restricted by integer multiples of π, depending on the mode order. In the case of general paraxial resonators, which may be composed of mirrors and lenses, and weakly guiding refractive index distributions of finite length, $\tilde{u}(x,y,z)$ can therefore be expected to be free of small-scale spatial oscillations with a scale length corresponding to the wavelength. This leads to the important conclusion that an efficient FEA discretization with a mesh size of the order of the wavelength or even much larger should be possible.

To take into account small fluctuations of the refractive index, we define

$$k = k_f - k_s(x,y,z) \tag{4}$$

where k_s generally is a complex valued quantity. Then, insertion of equations (2) and (4) into equation (1) delivers

$$-\Delta\tilde{u} + 2i(k_f - \delta)\frac{\partial\tilde{u}}{\partial z} + k_s\left(2k_f - k_s\right)\tilde{u} = \delta(2k_f - \delta)\tilde{u}\,. \tag{5}$$

In most cases $\delta / k_f <\approx 10^{-4}$, therefore the term δ in the first parenthesis on the left side and in the parenthesis on the right side usually can be neglected, delivering

$$-\Delta\tilde{u} + 2ik_f\frac{\partial\tilde{u}}{\partial z} + \left(k_f^{\,2} - k_s^{\,2}\right)\tilde{u} = \xi\,\tilde{u}\,. \tag{6}$$

Since we are looking for eigenmodes, the solutions $\tilde{u}$ and its derivatives with respect to z must meet the condition of periodicity imposed by the length $\tilde{L}$ of the substructure, i. e.

$$\tilde{u}(x,y,\tilde{L}) = \tilde{u}(x,y,0)\ , \tag{7a}$$

$$\frac{\partial}{\partial z}\tilde{u}(x,y,\tilde{L}) = \frac{\partial}{\partial z}\tilde{u}(x,y,0)\,. \tag{7b}$$

Perpendicular to the propagation direction the laser modes usually strongly decay with distance from the axis, but since we are forced to use a finite computational volume, a non-reflecting boundary condition must be used for the transverse evanescent part of the wave. An appropriate choice is to use a Robin boundary condition

$$\frac{\partial\tilde{u}}{\partial\vec{n}} - iC_b\tilde{u} = 0\,, \tag{8}$$

where $\partial/\partial\vec{n}$ denotes the derivation in direction of the normalized outer normal and C_b can be chosen as $C_b = k_f$.

Equations (6 - 8) describe a solvable eigenvalue problem for the cavity modes. To derive these equations only minor neglects have been made. The second derivative of $\tilde{u}$ with respect to z has not been neglected, as it is necessary to derive the paraxial wave equation. Therefore, it is expected that the FEA solution delivers results of high accuracy for the eigenmodes of the laser cavity.

3. Numerical Results

To verify the obtained numerical results we selected configurations that allow for application of the gaussian mode algorithm, since this is the only method permitting an exact analytical analysis of laser cavities. But of course, our method is not confined to those cases, but can be applied to any other cavity configuration described by equation (1).

Fig. 1 shows results for an empty cavity 1.0 mm long with a concave left end mirror with 5 mm radius of curvature and a right planar mirror. The diagram shows the spot size as a function of z obtained by our FEA approach in comparison with the gaussian spot size computed by the use of LASCAD™[5]. The width of the computational domain was 0.2 mm; in the computation 41 x 41 x 61 ≈ 102000 nodes have been involved. For the same case, Fig. 2

shows the Guoy phase shift along the resonator axis. Fig. 3 shows a 3D plot of the TEM_{22}-Mode in a long resonator obtained by FEA.

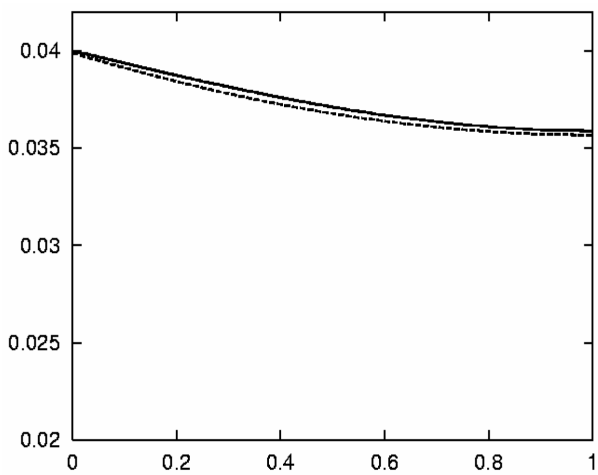

Fig. 1. Spot size along the axis of an empty with a concave left end mirror with 5 mm radius of curvature and a right planar mirror, FEA results - solid line, gaussian spot size - dashed line.

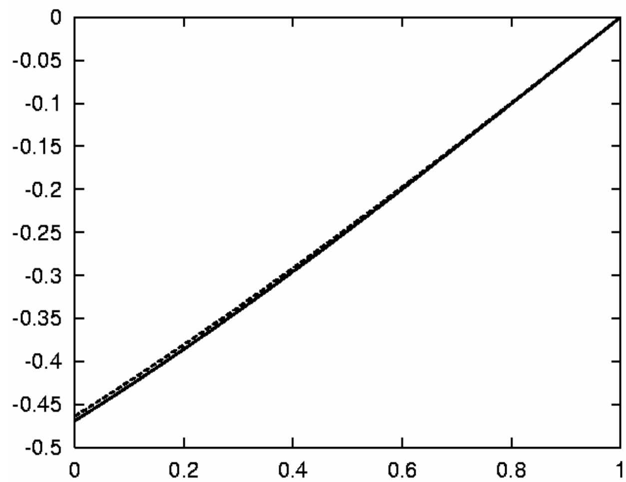

Fig. 2. Guoy phase shift along the axis of the cavity described in Fig. 1

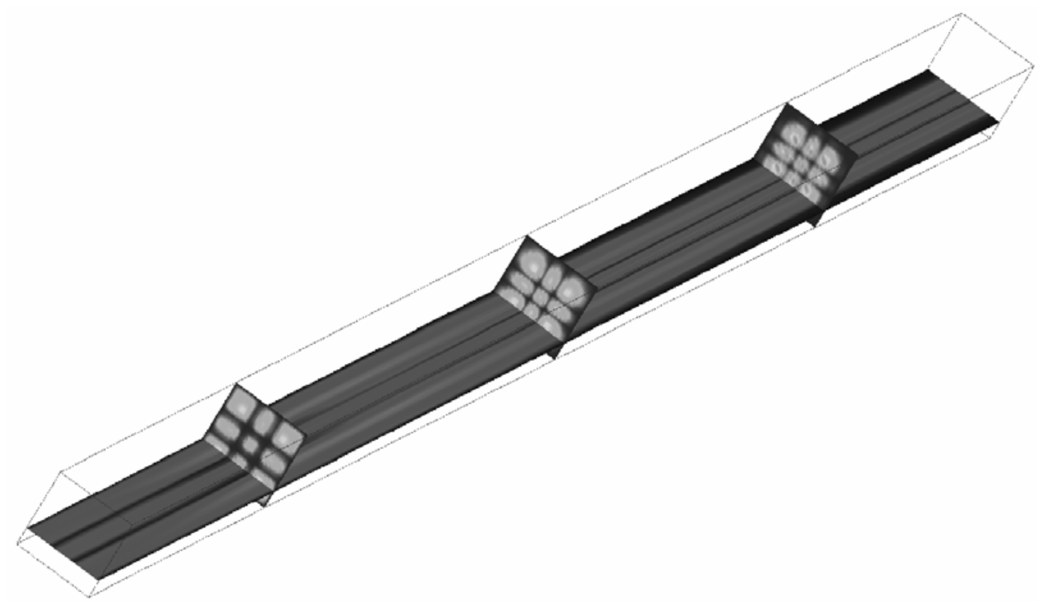

Fig. 3. TEM_{22}-Mode in a long resonator obtained by FEA

4. Conclusions

The presented results show that for gaussian cavities not only the spot size but also the Guoy phase shift obtained by our new FEA approach are in excellent agreement with the gaussian results. As a next step we are developing a combination our FEA approach for the electromagnetic field with FEA tools for the thermal and structural analysis of laser crystals, which already are available in our computer code LASCAD™[5], to realize an accurate and reliable numerical tool for the analysis of thermal lensing effects in SSL and DPSSL lasers.

5. References

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