Stability Diagram and Stability Criterions

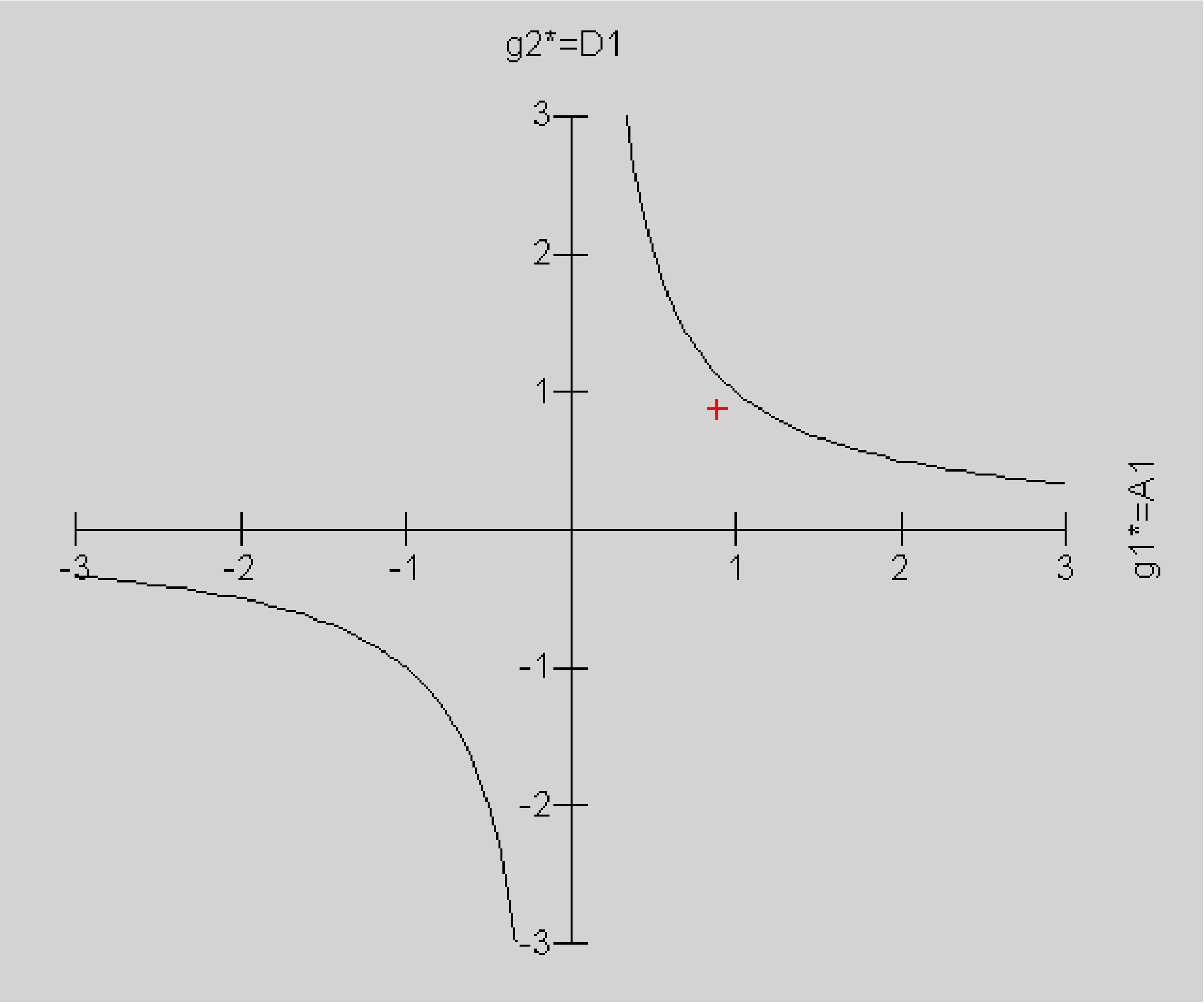
As described in textbooks for instance LASERS of A. E. Siegman, the stability diagram initially has been developed for the standing wave two-mirror resonator. For this resonator real and finite solutions of the gaussian beam parameters and spot sizes only exist, if the following criterion is met

$$0 \leq g_1 \, g_2 \leq 1$$

with

$$g_1 \equiv 1 - \frac{L}{R_1} \quad \text{and} \quad g_2 \equiv 1 - \frac{L}{R_2}$$

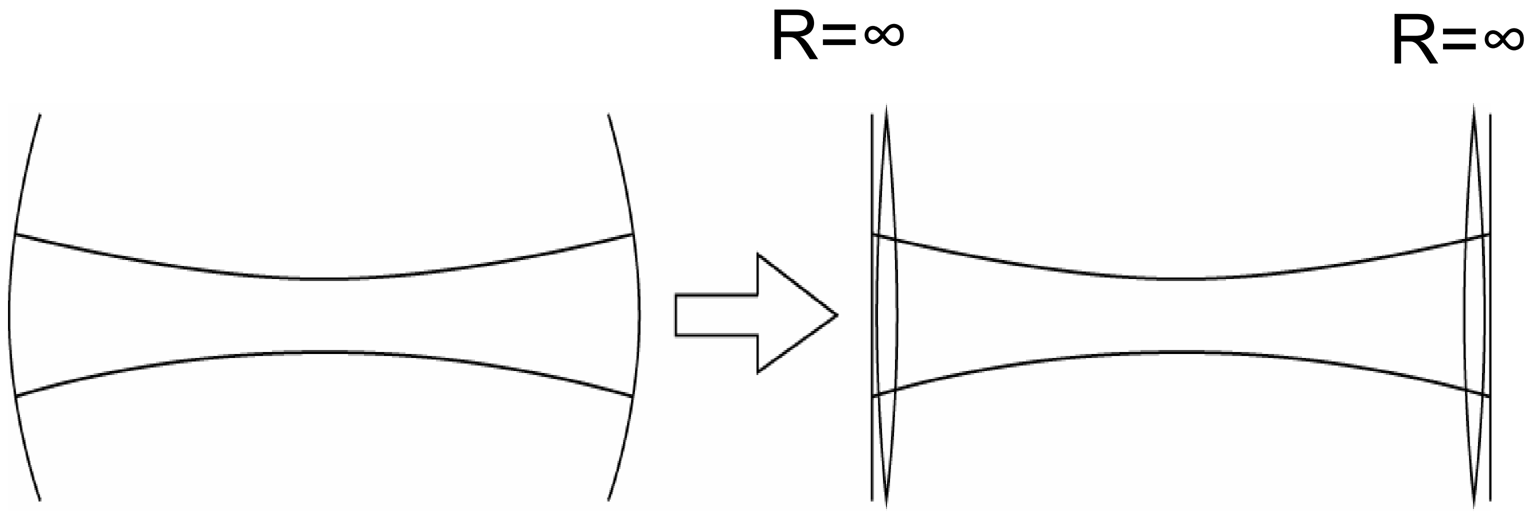
Here L is the distance of the mirrors and R_1 and R_2 are the radii of curvature of left and right mirror, respectively.



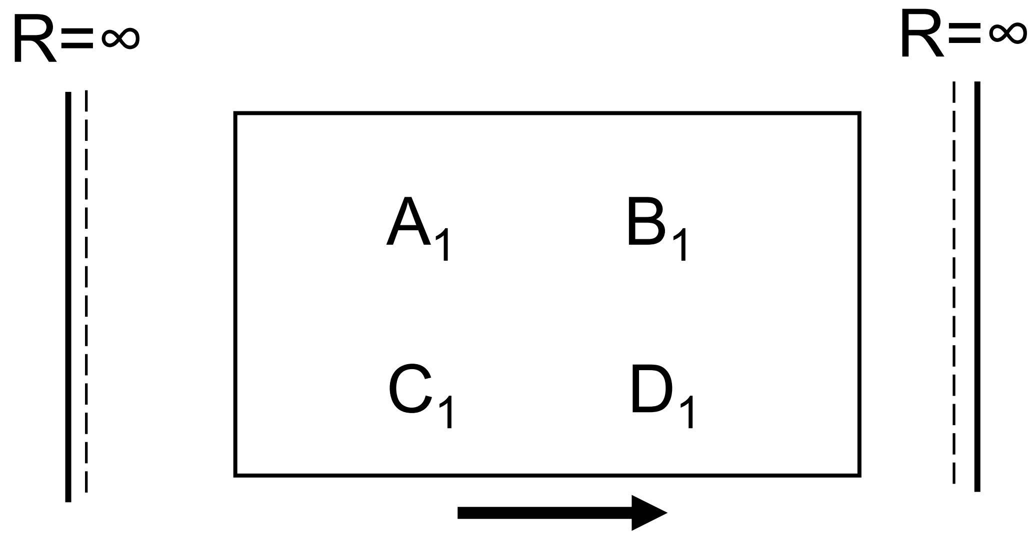
Stability diagram

As shown in PRINCIPLES OF LASERS, Sect. 5.5 by O. Svelto this concept can be generalized to the case of the general standing wave resonator with internal optical elements by introduction of the single-pass ABCD matrix.

For this purpose we replace the curved mirrors by plane mirrors combined with appropriate thin lenses



The elements between the plane mirrors now can be described by a single-pass ABCD matrix.



In case of an empty two-mirror resonator it can be shown that

$$g_1 = A_1 \qquad\qquad g_2 = D_1$$

For the general single-pass matrix representing a series of arbitrary internal elements these relations can be generalized to

$$g_1^* = A_1 \qquad\qquad g_2^* = D_1$$

The generalized parameters g^*_1 and g^*_2 must meet the same stability criterions as g_1 and g_2

$$0 \le g_1^* g_2^* \le 1$$

To derive this relation the full round-trip matrix has to be computed

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} D_1 & B_1 \\ C_1 & A_1 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} 2A_1 D_1 - 1 & 2B_1 D_1 \\ 2A_1 C_1 & 2A_1 D_1 - 1 \end{bmatrix}$$

Building the half-trace of the matrix at the right hand side we obtain

$$A_1 D_1 = \frac{1}{2}\left(\frac{A+D}{2} + 1\right)$$

Since the half-trace of the full round-trip matrix must mett the stability criterion

$$-1 \leq \frac{A+D}{2} \leq 1$$

it follows

$$0 \leq A_1 D_1 \leq 1$$

or

$$0 \leq g_1^* g_2^* \leq 1$$

Therefore, by the use of the generalized g-parameters a stability diagram can be shown for any standing wave resonator for instance with internal thermal lens.

Strictly speaking, the concept of the single-pass matrix only is valid if gain guiding can be neglected. However, since in most real cases the influence gain guiding is week, the generalized stability diagram is a useful tool to analyze to analyze cavity stability.

However, this concept cannot be used for ring cavities, since in this case the single-path matrix is identical to the full round-trip matrix. In this case the half-trace of the full round-trip matrix must be used as a reference value.

Alternatively, the concept of perturbation eigenvalues as introduced in the book *LASERS* of Siegman can be used

$$\lambda_a, \lambda_b = \frac{A+D}{2} \pm \sqrt{\left(\frac{A+D}{2}\right)^2 - 1}$$