Gaussian Beam Propagation Code

ABCD Matrices

Beam Propagation through a series of parabolic optical elements can be described by the use of ABCD matrices

Examples: Matrices for a mirror,lens, dielectric interface

$$M_{Mirror} = \begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix} \qquad M_{Lens} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

$$M_{\text{Curved dielectric interface}} = \begin{bmatrix} 1 & 0 \\ (n_2 - n_1)/R & 1 \end{bmatrix}$$

$$M_{Free\,Space} = \begin{bmatrix} 1 & L/n_0 \\ 0 & 1 \end{bmatrix}$$

The ABCD matrix algorithm can be applied on a propagating ray as well as on a propagating gaussian beam

Application on a ray $\vec{r}_0$ defined by position and slope

$$\vec{r}_1 = M \vec{r}_0$$

Siegman, LASERS, Chapt. 15,

Ray Optics and Ray Matrices

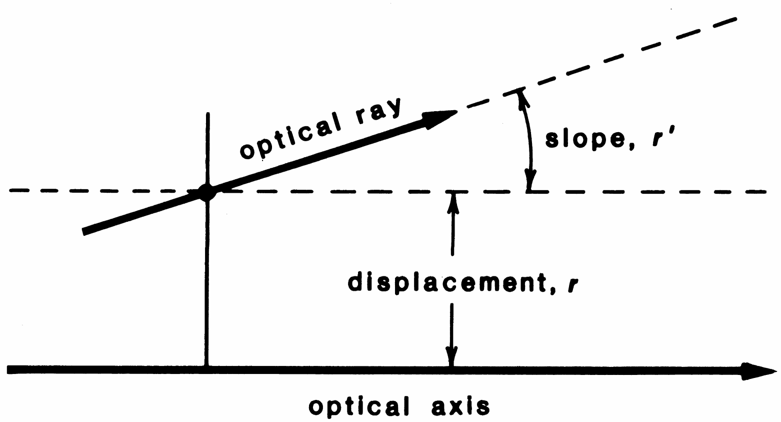


FIGURE 15.1
Definition of an optical ray.

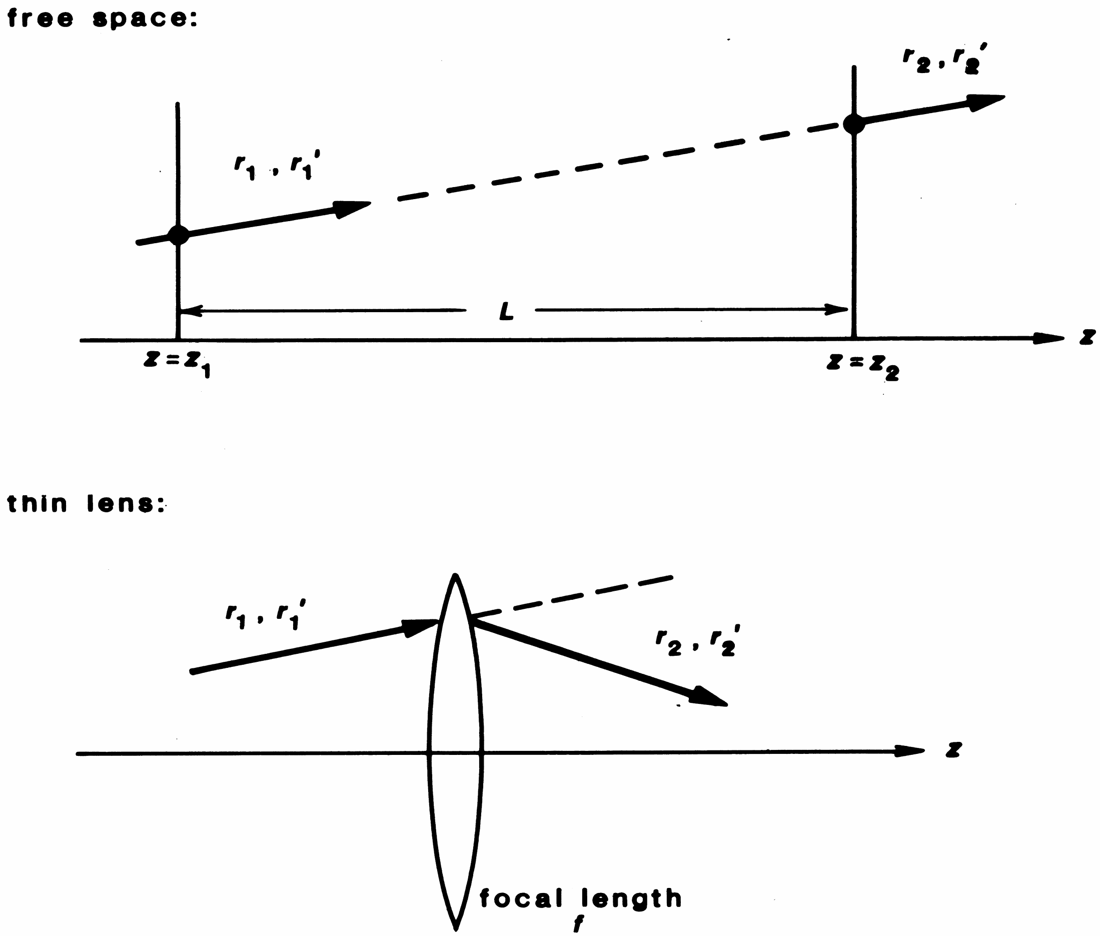


FIGURE 15.2
Optical-ray transformations through free space and through a thin lens.

Gaussian Paraxial Wave Optics

The ABCD matrix can also be applied to transform the so called q Parameter of a Gaussian beam

$$u(x,y,z) = \frac{1}{n_0\, q(z)} \exp\left[-jk \frac{x^2+y^2}{2R(z)} - \frac{x^2+y^2}{w^2(z)} \right]$$

R   radius of phase front curvature

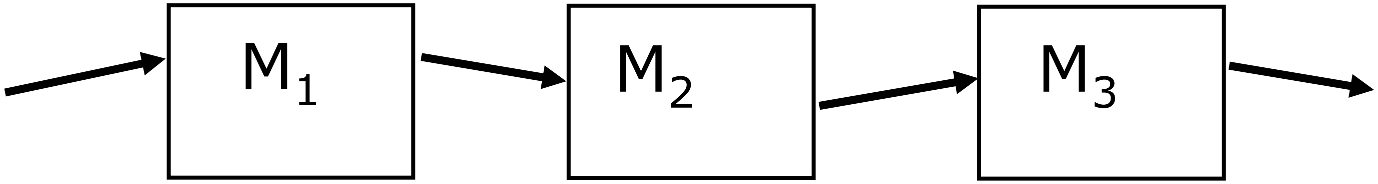
w   spot size defined as 1/e^2 radius of intensity distribution

The q parameter is given by

$$\frac{1}{q} = \frac{n_0}{R} - j\frac{\lambda_0}{\pi\ w^2}$$

Transformation of the q parameter by an ABCD matrix

$$q_2 = \frac{A\,q_1 + B}{C\,q_1 + D}$$



Ray Matrix System in Cascade

Total ray matrix

$$M_{tot} = M_n \; M_{n-1} \ldots M_2 \; M_1$$

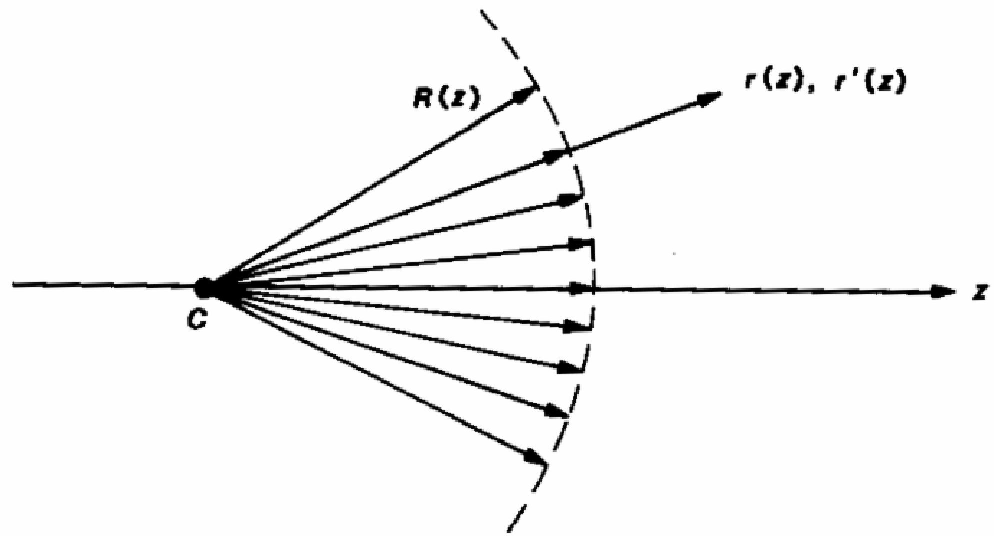


FIGURE 15.10
Spherical wave as a fan of rays.

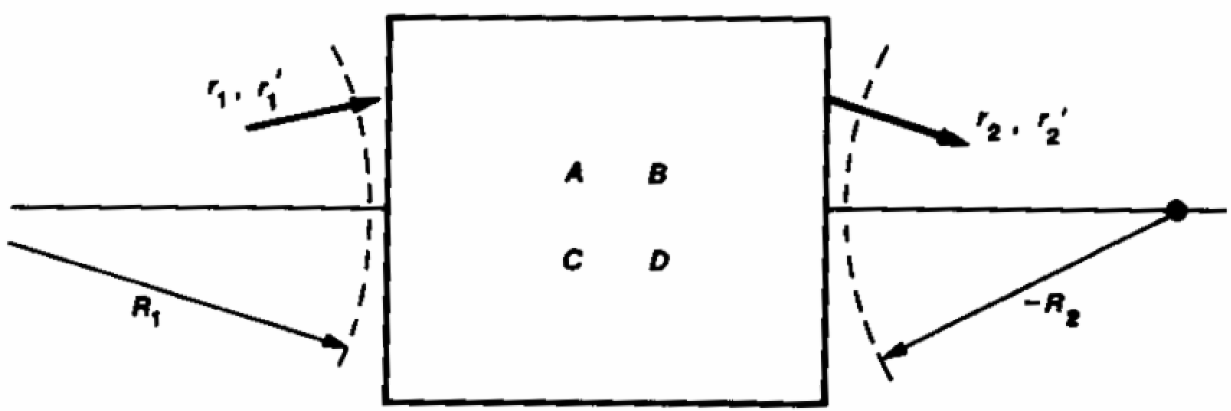
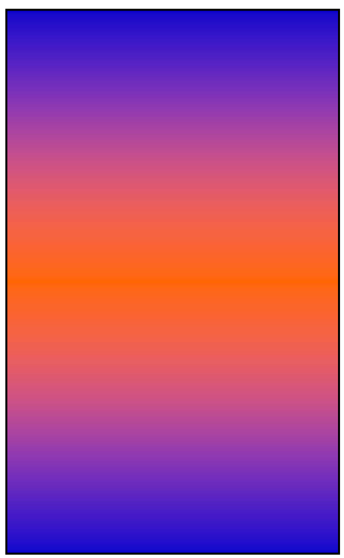


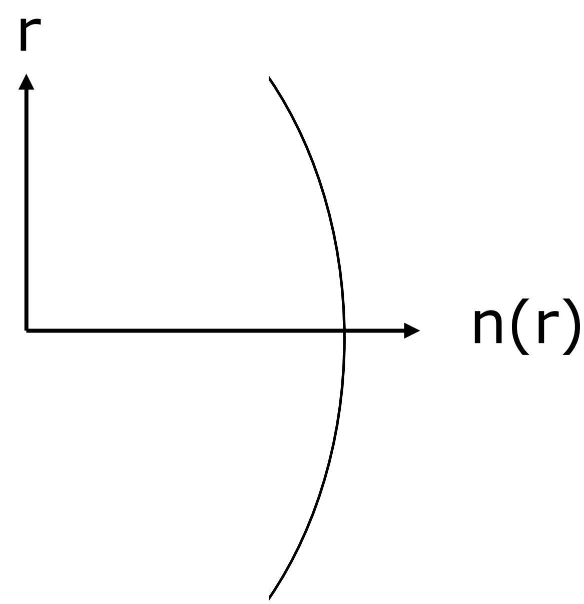
FIGURE 15.11
Spherical wave transformation through an arbitrary paraxial system.

Gaussian Duct

A. E. Siegman, LASERS

A gaussian duct is a transversely inhomogeneous medium in which the refractive index and the absorption coefficient are defined by parabolic expressions





Parabolic parameters n_2 and α_2 of a gaussian duct

$$n(x) = n_0 - \frac{1}{2} n_2 \, x^2$$

and

$$\alpha = \alpha_0 - \frac{1}{2} \alpha_2 \, x^2$$

n_2 parabolic refractive index parameter

α_2 parabolic gain parameter

ABCD Matrix of a Gaussian Duct

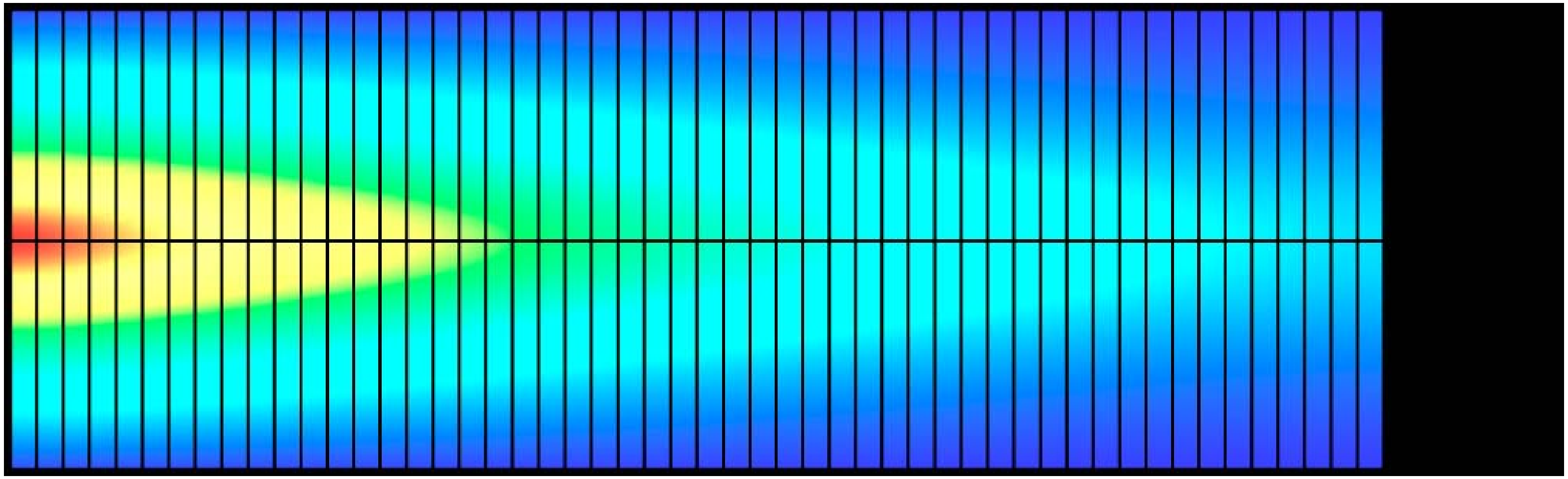
With the definition

$$\gamma^2 = \frac{n_2}{n_0} - j\,\frac{\lambda_0\,\alpha_2}{2\,\pi\,n_0}$$

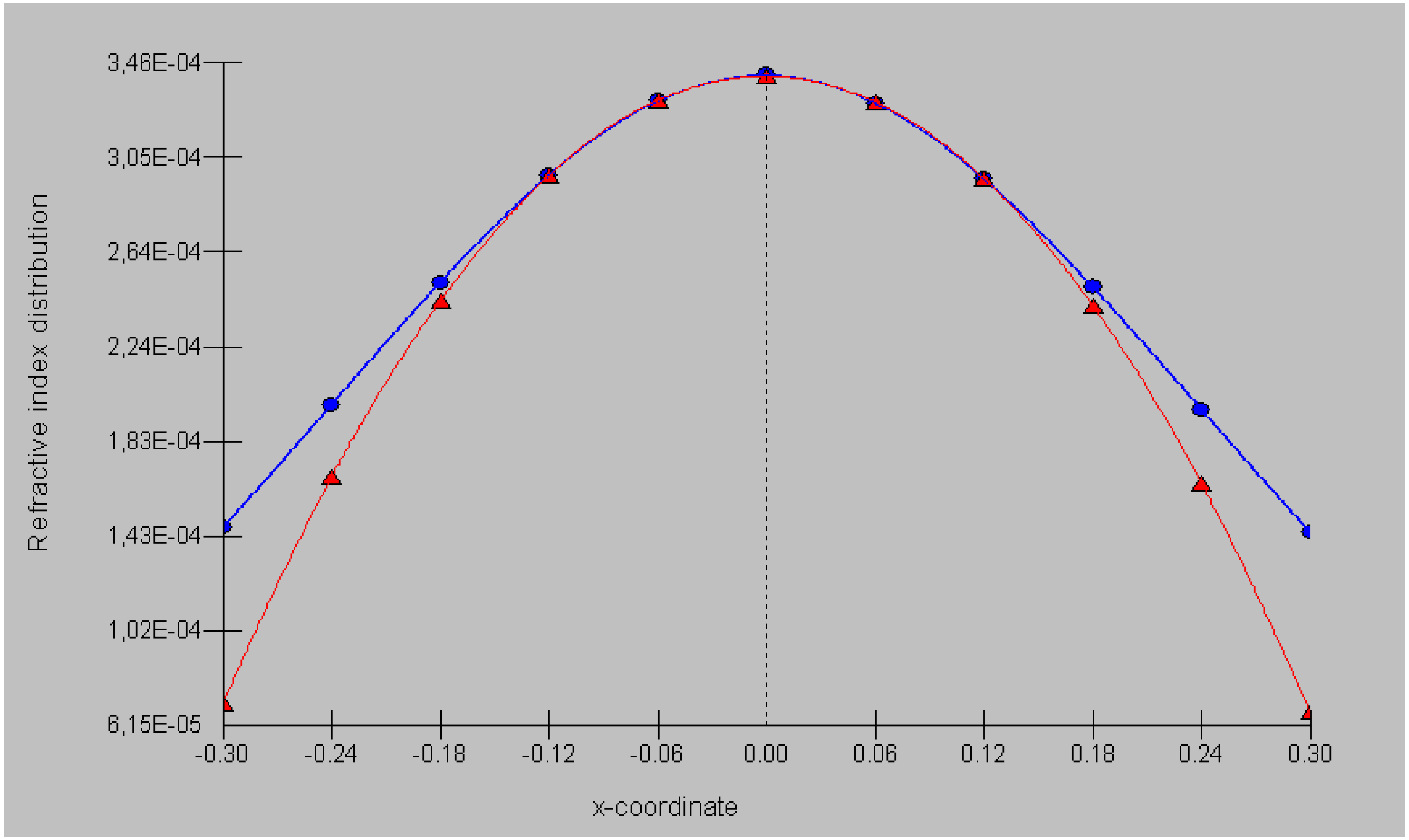
the ABCD matrix of a gaussian duct can be written in the form

$$\begin{bmatrix} A(z) & B(z) \\ C(z) & D(z) \end{bmatrix} = \begin{bmatrix} \cos\gamma(z - z_0) & \sin\big(\gamma(z - z_0)\big)/(n_0\gamma) \\ -n_0\,\gamma\sin\gamma(z - z_0) & \cos\gamma(z - z_0) \end{bmatrix}$$

In LASCAD the concept of the Gaussian duct is used to compute the thermal lensing effect of laser crystals. For this purpose the crystal is subdivided into short sections along the axis, and every section is considered to be a Gaussian duct.



A parabolic fit is used to compute the parabolic parameters for every section.



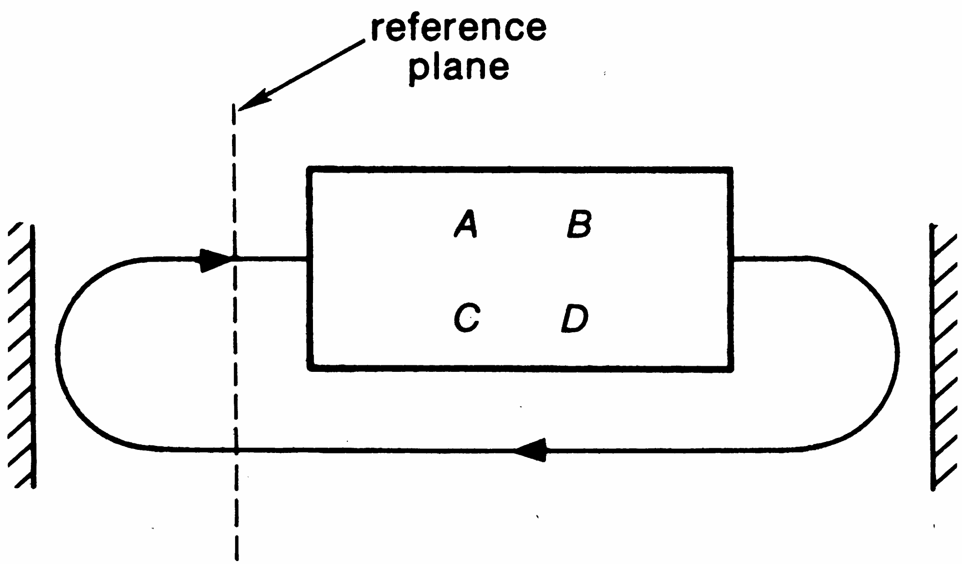
Example: Parabolic fit of the distribution of the refractive index

With the ABCD matrices of mirrors, lenses, internal dielectric interfaces, and Gaussian ducts most of the real cavities can be described.

To compute the eigenmodes of a cavity the q parameter must be self-consistent, that means it must meet the round-trip condition.

Round-Trip Condition

$$q_2 = \frac{A q_1 + B}{C q_1 + D} = q_1$$



The round-trip condition can be used to derive a quadratic equation for the q parameter.

$$\frac{1}{q_a},\frac{1}{q_a} = \frac{D-A}{2B} \mp \frac{1}{B}\sqrt{\left(\frac{A+D}{2}\right)^2 - 1}$$

All these computations are simple algebraic operations and therefore very fast.

Gaussian Optics of Misaligned Systems

With 2 x 2 ABCD Matrices only well aligned optical systems can be analyzed. However, for many purposes the analysis of small misalignment is interesting.

This feature has not been implemented yet the LASCAD program, but it is under development, and will be available within the next months.

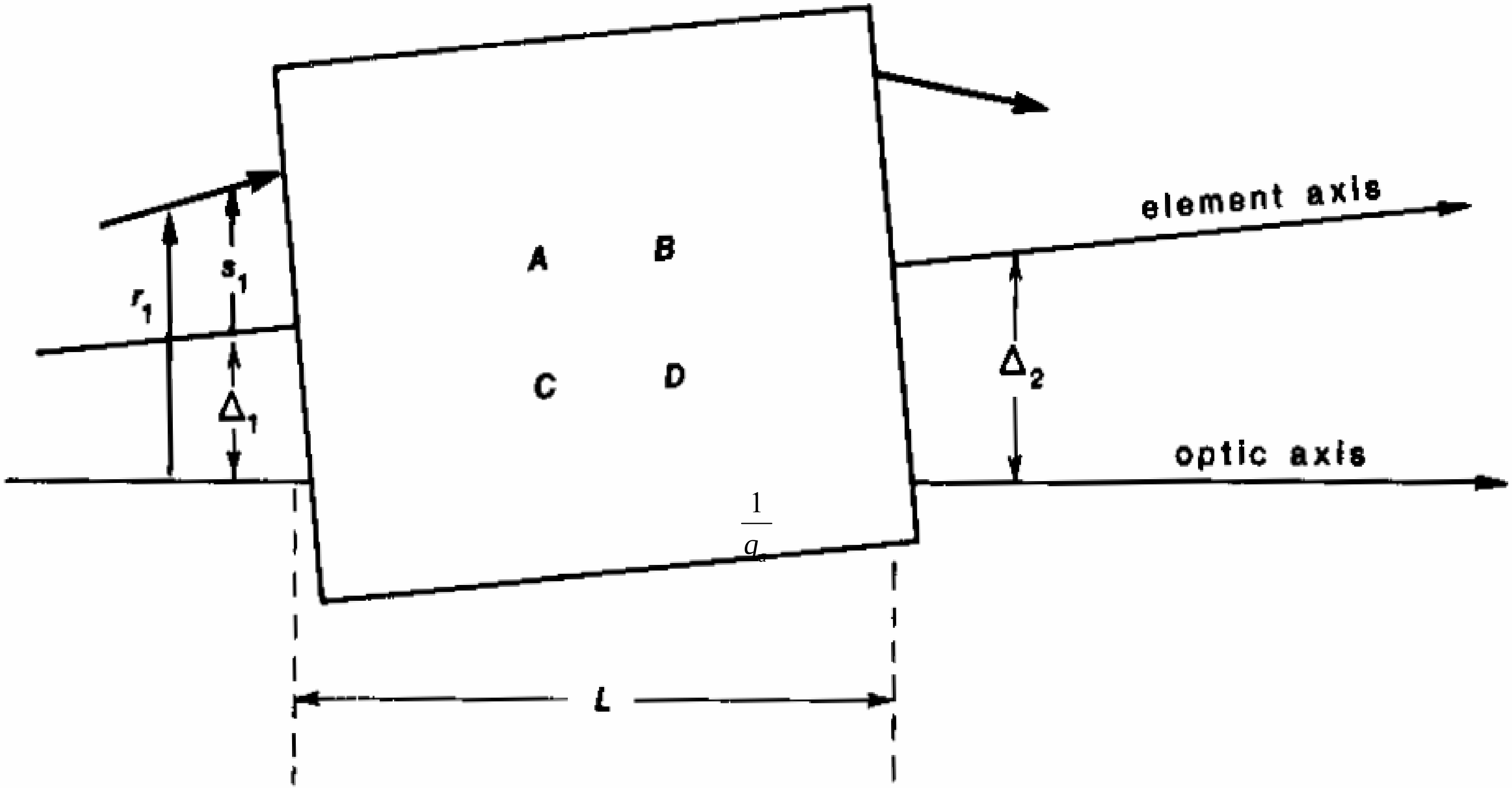


FIGURE 15.19
Notation for analyzing a misaligned paraxial optical element.

As shown in the textbook LASERS of Siegman the effect of misalignments can be described by the use of 3x3 matrices

$$\begin{bmatrix} r_2 \\ r_2' \\ 1 \end{bmatrix} = \begin{bmatrix} A & B & E \\ C & D & F \\ 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} r_1 \\ r_1' \\ 1 \end{bmatrix}$$

Here E and F are derived from the parameters $\Delta_{1(2)}$ describing the misalignmet of the element

These 3x3 Matrices also can be cascaded to describe the propagation of a gaussian beam through any sequence of cascaded, and individually misaligned elements.

$$\vec{r}_N = M_{tot}\,\vec{r} + \vec{E}_{tot}$$

M_{tot} is the total ABCD Matrix

$\vec{E}_{tot}$ is the total misalignment vector which depends on the individual misalignments and the individual ABCD matrices