Dynamic Analysis of Multimode and Q-Switch Operation (DMA)

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The Dynamic Multimode Analysis (DMA) uses transverse eigenmodes obtained by the gaussian ABCD matrix approach, to provide a time dependent analysis of multimode and Q-switch operation of lasers.

For this purpose the transverse mode structure in the cavity is approximated by a set of M Hermite-Gaussian (HG) or Laguerre-Gaussian (LG) modes.

Since HG and LG modes are representing **sets of orthogonal eigenfunctions** with different eigenfrequencies, it is assumed that **each transverse mode oscillates independently**, and therefore the influence of short-time mode locking and interference effects between the modes can be neglected on average.

Multimode Rate Equations

$$S_C = \sum_{i=1}^{M} S_i \qquad \text{i=1,...,M}$$

$$\frac{\partial S_i}{\partial t} = \frac{c\sigma}{n_A} \int_{\Omega_A} N S_i s_i dV - \frac{S_i}{\tau_C}$$

$$\frac{\partial N}{\partial t} = -\frac{c\sigma}{n_A} N\, S_C s_C - \frac{N}{\tau_f} + R_p \frac{N_{dop} - N}{N_{dop}}$$

$S_i(t)$ number of photons in transverse mode i
$S_C(t)$ total number of photons in the cavity

$s_{i,C}(x,y,z)$ normalized density distribution of photons

n_A	refractive index of the active medium
c	vacuum speed of light

$N(x,y,z,t) = N2 - N1$ population inversion density ($N1 \sim 0$)

$R_P = \eta_P P_a / h\nu_P$	pump rate
η_P	pump efficiency
$P_a(x,y,z)$	absorbed pump power density
σ	effective cross section of stimulated emission
τ_C	mean life time of laser photons in the cavity,
τ_f	spontaneous fluorescence life time of upper laser level
N_{dop}	doping density.

$\widetilde{L}$

An important quantity is the mean life time τ_C of the laser photons in the cavity. It is given by

$$\tau_c = \frac{t_{rtrip}}{L_{\mathrm{Res}}} = \frac{2\widetilde{L}}{c(L_{roundtrip} - \ln(R_{out}))}$$

where

$\widetilde{L}$ optical path length of the cavity

t_{rtrip} period of a full roundtrip of a wavefront

$L_{roundtrip}$ round trip loss

R_{out} reflectivity of output mirror

A detailed theoretical description of the DMA code is given in the LASCAD manual Sect. 7.

In the following am only giving a comprehensive description of the main features.

To obtain the normalized photon densities s_i (i=C; 1,…,M) the complex wave amplitudes $u_i(x,y,z)$ are normalized over the domain $\Omega=\Omega_{2D}x[0,LR]$ of the resonator with length L_R. Here the u_i (i=1,…,M) denote the amplitudes of the individual modes, whereas u_C denotes the amplitude of the superposition of these modes In our incoherent approximation absolute square of this superposition is given by

$$\left|u_C(x,y,z)\right|^2 = \sum_{i=1}^{M}\left|u_i(x,y,z)\right|^2$$

The amplitudes u_i and the normalized photon distributions s_i are connected by the following relation

$$s_i = \begin{cases} \dfrac{n_A}{V_i}\left|u_i\right|^2 & \text{inside the crystal} \\ \dfrac{1}{V_i}\left|u_i\right|^2 & \text{outside the crystal} \end{cases}$$

Note that the photon density inside the crystal is by a factor n_A higher than outside due to the reduced speed of light.

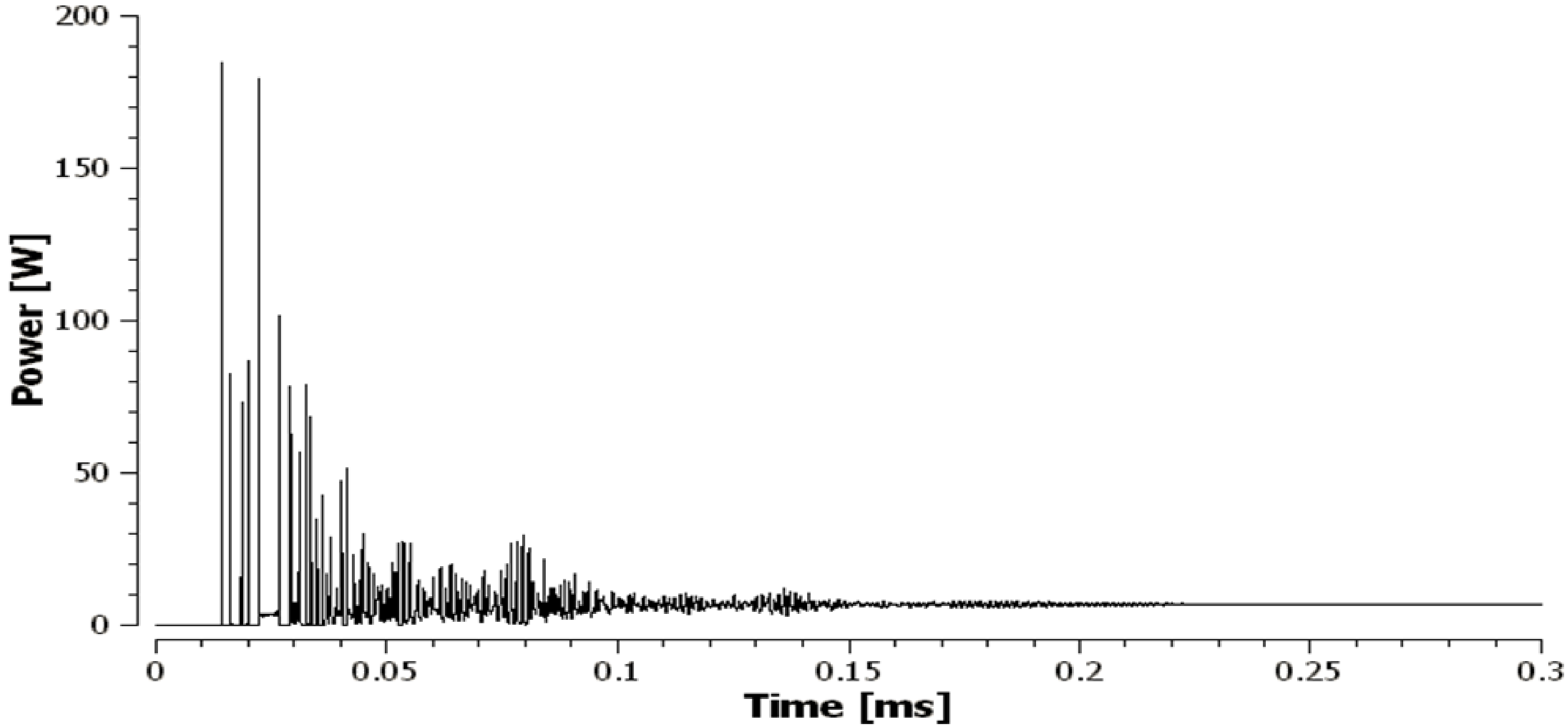
Laser Power Output

The laser power output is obtained by computing the number of photons passing the output coupler per time unit. In this way one obtains for the power output delivered by the individual transverse modes

$$P_{i,out}(t) = h\nu_L S_i(t) \frac{-\ln(R_{out})}{t_{rtrip}} \left[1 - 0.5\ln(R_{out})\right]$$

R_{out} reflectivity of output mirror

t_{rtrip} period of a full roundtrip of a wavefront



This plot shows a typical time dependence obtained for the total power output.

Since the computation starts with population inversion density $N(x,y,z,t)=0$, a spiking behavior can be seen at the beginning, which attenuates with increasing time.

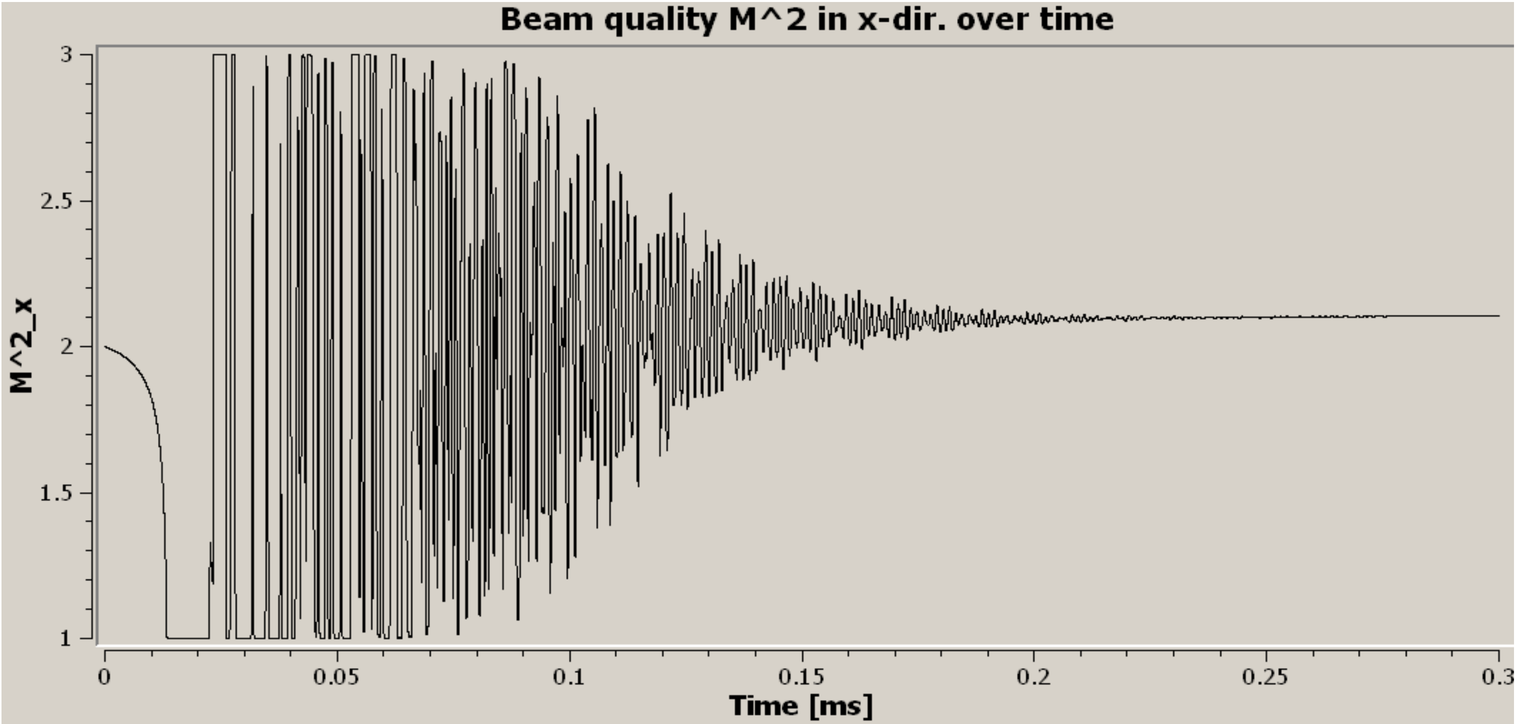
Beam Quality M^2

The beam quality factors M_x^2 and M_y^2 are computed according to Siegman and Townsend using the expressions

$$M_x^2(t) = \sum_{i=1}^{M} \left(2p_i + 1\right) c_i(t)$$

$$M_y^2(t) = \sum_{i=1}^{M} \left(2q_i + 1\right) c_i(t)$$

Here p_i and q_i are the transverse mode orders of the i-th gaussian mode in x- and y-direction, respectively. The coefficients $c_i(t)$ are the relative contributions of the inividual modes to the total power output.



This plot shows a typical time dependence obtained for the beam quality.

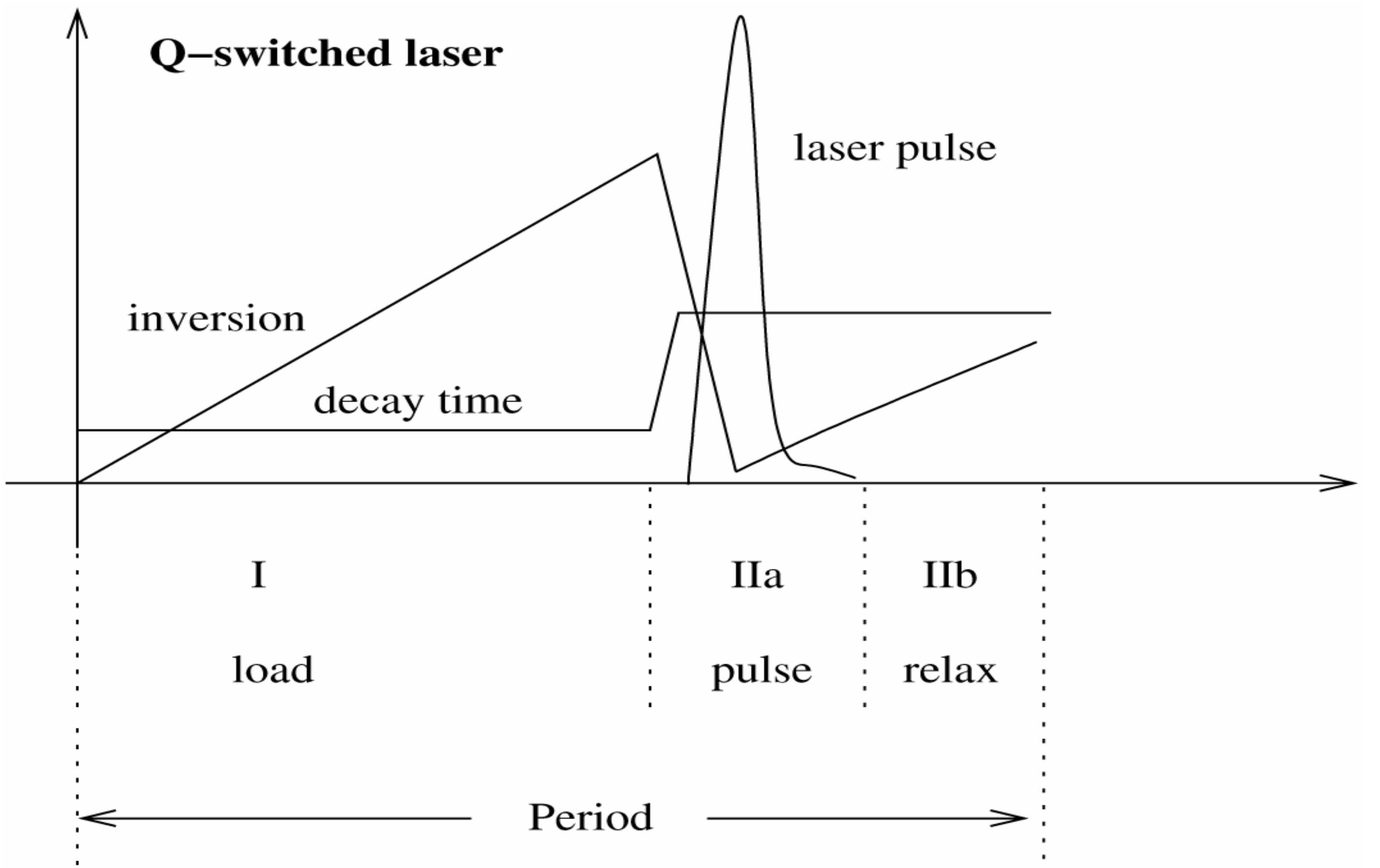
Again the spiking at the beginning is caused by the vanishing inversion density N(x,y,z,t) at the start of the computation.

Modeling of Q-Switch Operation

Time dependence of active Q-switching is characterized by three time periods which can be described as follows:

- load period – period
- pulse period – period Iia
- relaxation period – period IIb

Development of population inversion and laser power during these periods is shown schematically in this plot



During the load peiod, it is assumed that the photon number in the individual modes vanishes. This simplifies the rate equations to

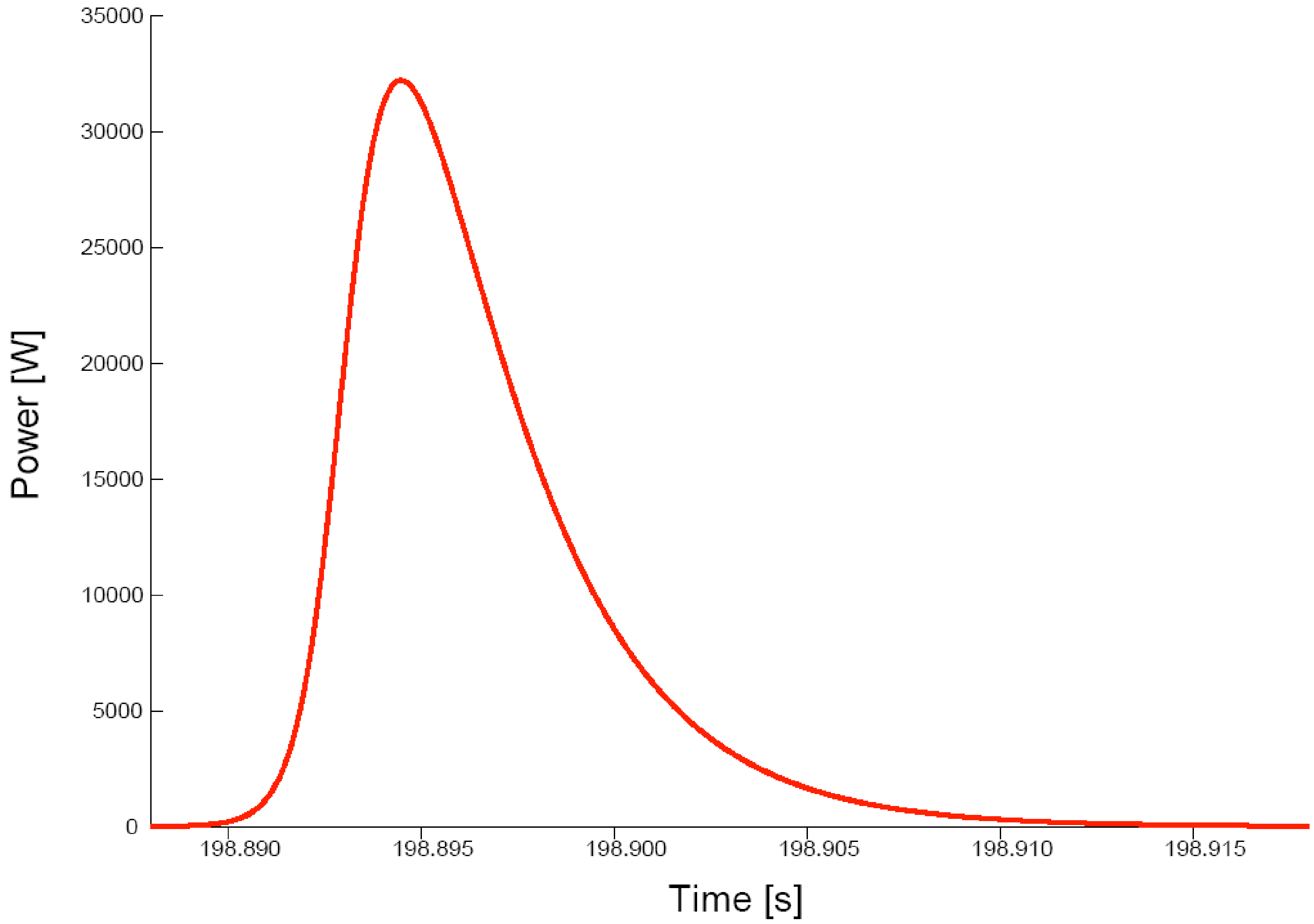
$$\frac{\partial S_i}{\partial t} = 0, \qquad \text{i=1,...,M}$$

$$\frac{\partial N}{\partial t} = -\frac{N}{\tau_f} + R_p \frac{N_{dop} - N}{N_{dop}}$$

To prevent lasing during the load period a high artificial intra-cavity loss is introduced

After the load period this artificial loss is removed that means the Q-switch is opend and the pulse can develop.

A typical pluse shape ontained with our DMA code is shown on the next slide.



Typical pulse shape computed with the DMA Code

Apertures and Mirrors with Variable Reflectivity

Apertures and output mirrors with variable reflectivity can be taken into account in the Dynamic Multimode Analysis by introducing specific losses L_i for the individual modes.

This leads to mode specific mean life times $\tau_{C,i}$ of the photons due to mode specific losses.

In case of an aperture with radius R_A at position z_A and a mirror with uniform reflectivity, the mode specific losses are described by

$$L_i = L_{rtrip} - \ln\left[R_{out} \int_0^{R_A} \int_0^{2\pi} \tilde{s}_i(r, z_A) 2\pi \, dr d\varphi \right]$$

Here $\tilde{s}_i(r, z_A)$ is the photon distribution of mode i at position z_A normalized with respect to the transverse coordinates

In case of a mirror with variable reflectivity, for instance a gaussian mirror, the mode specific losses are described by

$$L_i = L_{rtrip} - \ln\left[\int\int R_{out}(x,y)\tilde{s}_i(x,y,z_A)dxdy\right]$$

where R_{out} is a function is a function or transverse coordinates x and y.

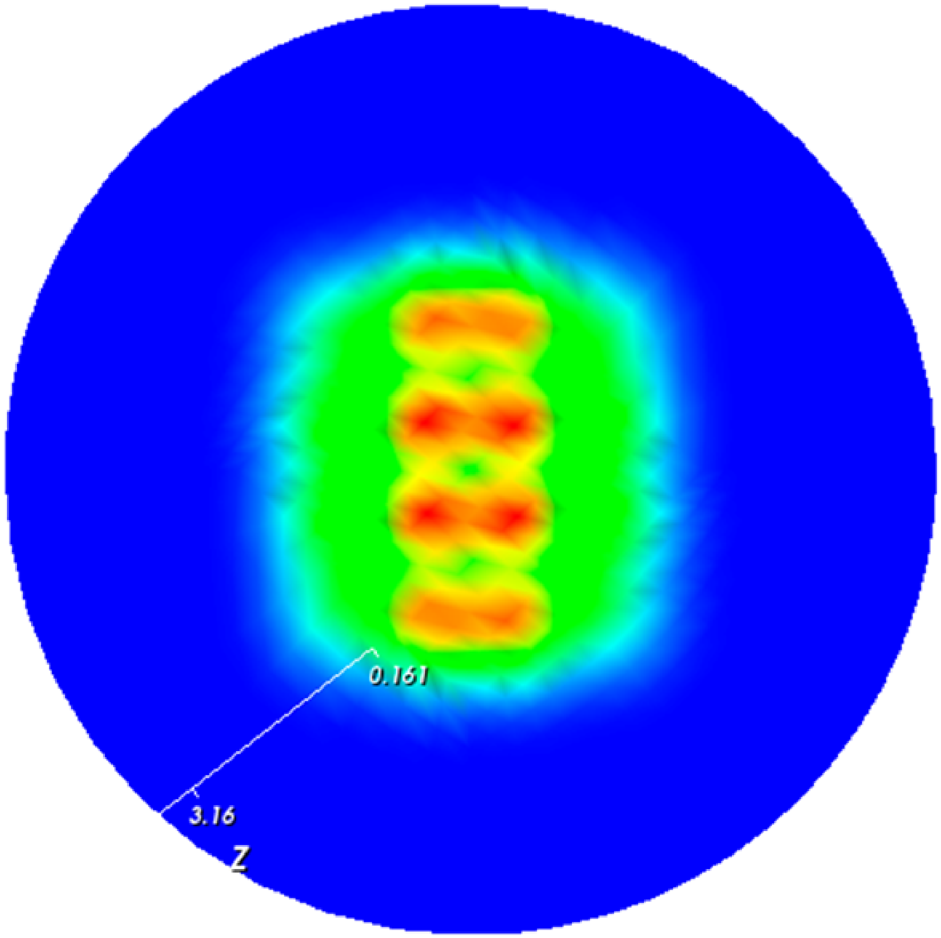
An important realisation of mirrors with variable reflectivity are supergaussian output mirrors. The reflectivity of such mirrors is described by

$$R(x,y) = R_0 \exp\left(-2\left|\frac{x}{w_{trx}}\right|^{SG} -2\left|\frac{y}{w_{try}}\right|^{SG} \right) + R_{\min}$$

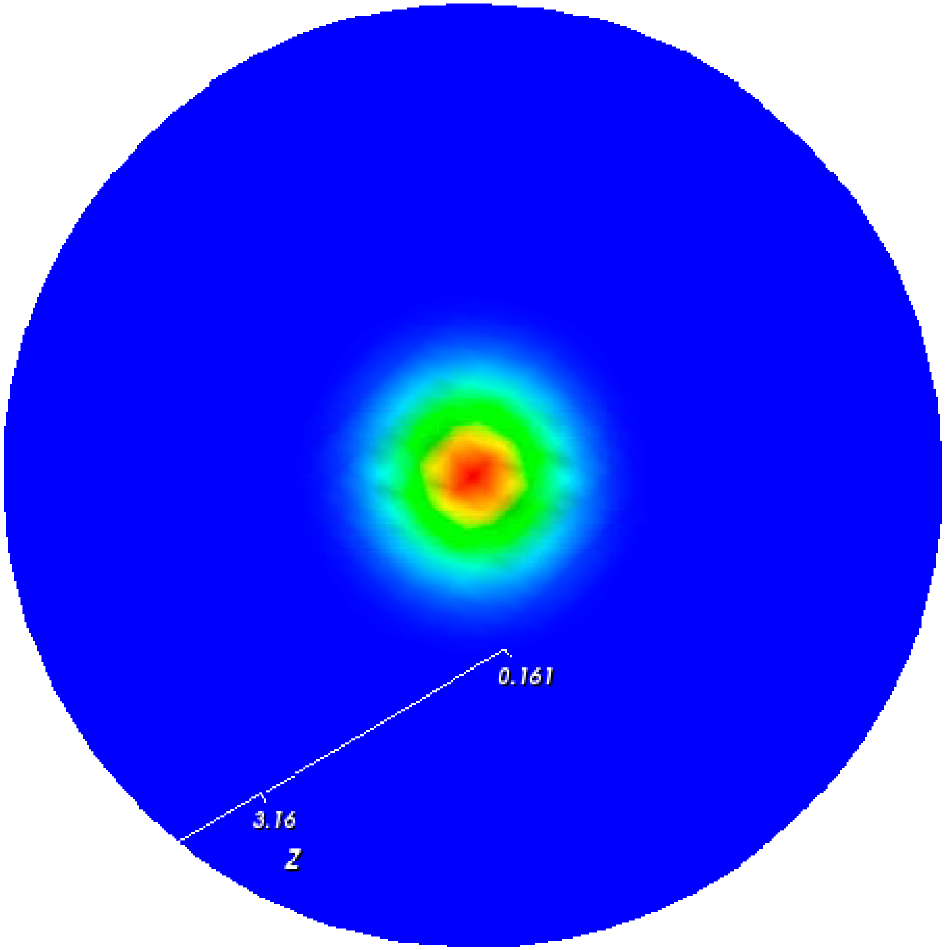
Here R_{min} is a peripheral bottom reflectivity.

With supergaussian mirrors the beam quality can be improved considerably without loosing too much power output. This shall be demonstrated.

This shall be demonstrated by the following example.



Beam profile without confining aperture.
Power output 6.87 W



Beam profile for the same configuration with supergaussian aperture. Power output 4.22 W