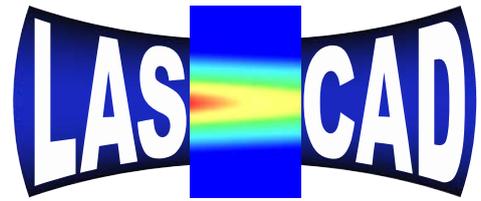


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LASer Cavity
Analysis & Design

LASCAD™ 3.6 Manual

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1 System Requirements

Supported operating systems: Windows 2000/XP/Vista/Windows 7 and 8

Minimum screen resolution: 1024 x 768 pixels

Main Memory (RAM): 256 to 512 MB is the recommended minimum.

FEA computations: If the FEA model does not fit into the main memory, Windows allocates virtual memory on the hard disk, which causes an extreme performance penalty. The following rule of thumb can be used to estimate memory consumption: If the FEA model has $n * 1000$ elements, you should have at least $2 * n$ MB main memory. For multi-material models the memory usage increases further. See also section 6.10.6.

2 Installation

To install the software, insert the CD-ROM and wait for Setup to start automatically or run `setup.exe` manually.

In the directory `Documentation` on the CD-ROM you also will find the files `Quick_Tour.rtf`, `Manual.rtf`, `Demo_Limitations.rtf`, `brochure.pdf` and `article.doc`. The latter is a reprint of an article published in *Laser Focus World* in May 2000.

3 Physical units

The unit of length is always [mm] except for wavelength and spot size in which cases it is [μm]. Deviations from this rule and other units are specified explicitly in the following and in the program.

4 Computational Methods

4.1 Complex Gaussian Mode Algorithm

The gaussian mode algorithm of the program computes the mode structure in the laser cavity together with the external beam by the use of complex ABCD matrices as described e.g. in the textbook *A.E. Siegman, LASERS*, Chap. 20. For this purpose, all optical elements in the resonator must be approximated parabolically. Usually, surfaces of mirrors or dielectric interfaces are spherical, and therefore can be considered as parabolic approximately. Thermally induced distributions of the refractive index are fitted parabolically in x- and y-direction perpendicular to the optical axis at the FEA discretization points. The same holds for thermally deformed surfaces. This procedure yields sufficient accuracy in many cases. The program uses the second derivative of these fits, see section 6.13. Since the gaussian algorithm is delivering reliable approximation in many cases, it is fully integrated into the GUI of LASCAD, and is used during generation of a resonator configuration.

4.2 Finite Element Analysis (FEA)

Temperature distribution, deformation and stress of thermally loaded crystals are computed by the use of FEA. Since the number of configurations interesting in laser technology is confined, pre-designed models have been provided to facilitate the use of FEA for the laser engineer. Presently the following models are available: end pumped cylindrical rods and slabs, rods and slabs with approximate top hat pump light distribution, side pumped cylindrical rods and side pumped slabs, rods and slabs with numerical input of absorbed pump power density. Other models are under development, or can be delivered on request. These models can be adjusted without detailed knowledge of FEA model generation by the use of the LASCAD GUI as described in section 6.10. The dimensions of the crystal can be changed. The pump beam distribution can be modeled very flexibly by the use of super-gaussian functions. Different modes of surface cooling can be selected. Undoped regions of the crystal can be defined. By the use of a modern automatic mesh generator an equidistant grid is generated inside the crystal which is adjusted to the surfaces by irregular elements. The equidistant grid enables use of very fast solver technologies. A 3D visualization code based on OpenGL technology provides detailed display of FEA results.

4.3 Physical Optics Code based on the Beam Propagation Method (BPM)

For cases where physical optics effects are important the FEA results can alternatively be used as input for a physical optics code based on a FFT (Fast Fourier Transform) beam propagation method (BPM). Different from the gaussian algorithm this code is not using parabolic approximations, and therefore, is able to take into account the full 3D distribution of the refractive index and the structural deformation as obtained from FEA. For this purpose, a wave front is propagated in small steps through the crystal taking into account the FEA results at every step. The cavity configuration developed for the gaussian algorithm can be directly converted into an input for this code. The beam profile is computed by the use of a Fox and Li type iterative round-trip procedure which however involves more modern computational methods. To control convergence during the iteration process a graphics window is opened after starting BPM that shows the spot size as it develops with increasing number of cavity iterations. Simultaneously a second graphics window shows the intensity distribution at the output-coupling mirror after every round-trip. At the end of the iteration process also the phase profile can be displayed.

In addition, a numerical analysis of the Eigenmode structure is carried through. For this purpose the amplitude data are sampled at the right mirror for every round trip. By the use of these data an eigenfrequency analysis is carried through that uses the FFT of a field correlation function as shown in the paper of *M.D.Feit and J.A.Fleck, "Spectral approach to optical resonator theory"*, Appl. Opt., Vol.20, 2843-2851 (1981). Afterwards in a second iteration process the Eigenmode profiles are computed for the strongest peaks of the eigenfrequency spectrum.

5 Starting the Program

After starting LASCAD the dialog "Select Working Directory" appears asking you to define a working directory which will contain all files and directories created during a LASCAD session. After clicking "OK" the main LASCAD window appears. Click "File/New Project" in the menu bar of this window or the leftmost icon button in the tool bar or simply press the RETURN key. A dialog appears, with the options "Standing Wave Resonator", "Ring Resonator" and "External Beam" which are described in detail in section 6.4. To get started the first time, choose the "Standing Wave Resonator", and leave the number of face elements and the wavelength [μm] unchanged. After closing this dialog with "OK" or the RETURN key two additional windows appear. The upper one (**Mode Plot** window) shows the gaussian mode shape for a simple standing wave resonator with 2 mirrors, visualized by 5 lines. The upper line is showing the shape of the spot size along the axis, the other lines are provided to give the picture a nicer appearance. The lower window ("*Parameter Field*" window) shows a parameter field with the parameter values used in the computation. If you enter a new value into one of the number boxes of this window and click afterwards the apply button or use the RETURN key, a new gaussian mode computation is carried through. Details of these and the other LASCAD windows are described in the following. Many of the windows can be scaled by the use of the maximize and minimize buttons or by dragging their borders with the mouse. The windows are described sequentially in an order as a new user of LASCAD might open them.

If already a LASCAD project file *.lcd exists, you can start the LASCAD program by double clicking on this file in the explorer.

6 Windows of LASCAD

6.1 Window "Parameter Field"

The program uses mirrors, thin lenses or dielectric interfaces as face elements. The space between the elements is filled by a medium. Elements and media are described by parameters. Most of the parameters can be defined independently for the two directions perpendicular the beam axis. Confirmation of a parameter modification with the RETURN key starts recomputation of the gaussian mode and generates a new mode plot.

The first 3 rows of the "*Parameter Field*" window are showing the following items:

- **Element number:** Corresponds to the element number used in the mode plot window. The flashing number field corresponds to the flashing vertical line in the mode plot window. Clicking on a number field or on one of the vertical lines in the upper window sets the flashing to an other element.
- **Element Type:** The Drop-down boxes allow selecting from 3 types: mirror, dielectric interface (I-Face) and thin lens.
- **Distances between the elements:** The values in the boxes are identical with the distances shown in the mode plot window.

Below of the first 3 rows you see a tab control with the following 5 tabs:

- **x-Plane Parameters**
- **y-Plane Parameters**
- **Apertures**
- **Miscellaneous**
- **Spot Sizes.**

The related index cards contain groups of number boxes where you can enter parameter values.

6.1.1 Tab "x-Plane Parameters"

Row Angle of Incidence [degrees]: The angle of incidence defines the angle between the normal to the element and the axis of the beam incident from the left side. For a beam propagating in a medium with refractive index >1 , a warning sign pops up, if the angle between the beam coming from the left and the normal to the element exceeds the limiting angle of total refraction.

The angle of incidence is not reflected in the mode plot, which shows a rectilinear representation. Nevertheless, computation takes into account this parameter in the ABCD matrix algorithm. Presently, beam propagation must occur within the x-z-plane i.e. the plane defined by the propagation axis (z-axis) and the x-axis. A full 3-D propagation path is currently being integrated into the physical optics code.

Row Type-Parameter [mm]: This parameter depends on the type chosen in the row "Element Type" above. For mirrors and dielectric interfaces it represents the radius of curvature, for lenses the focal length.

To toggle between convex or concave curvature or between focusing and defocusing lenses, change the sign of the parameter. The orientation of the curvature is symbolized in the mode plot window; see section 6.2.

Row Refractive Index x-comp. [mm]: x-component of the refractive index of the medium.

Row Refractive Parameter x-comp. [1/mm²]: Second x-derivative of the distribution of the refractive index times (-1) (corresponds to n_2 in *A.E. Siegman, LASERS*, University Science Books, Chapt. 20.3 *)).

Row Gain Parameter x-comp. [1/mm²]: Second x-derivative of the distribution of the gain coefficient times (-1) (corresponds to $-\alpha_2$ in *A.E. Siegman, LASERS*, Loc. cit. *)).

*) In *A.E. Siegman, LASERS*, University Science Books, Chapt. 20.3 parabolic parameters n_2 for the refractive index and α_2 the loss coefficient are defined as follows

$$n(x) = n_0 - \frac{1}{2}n_2x^2 \quad \text{and} \quad \alpha(x) = \alpha_0 + \frac{1}{2}\alpha_2x^2$$

n_2 is identical to the Refractive Parameter in LASCAD, $-\alpha_2$ is identical to the Gain Parameter in LASCAD.

6.1.2 Tab "y-Plane Parameters"

The 4 rows of this index card correspond to the lower 4 rows of the Tab "x-Plane Parameters". After entering a value into one of these boxes different from the corresponding x-plane value, the background becomes blue indicating, that astigmatism is set for this parameter. The change of color also takes place in the corresponding x-plane box. If astigmatism is set, the corresponding x-plane value can be changed independently of the y-value, otherwise the y-plane value always follows the x-plane value. Press the CTRL key and click into one of the corresponding boxes to set the y-plane value back to the x-plane value.

6.1.3 Tab "Apertures"

This index card shows 2 rows where you can enter radii of light stops. Again you can use different values for x- and y-plane. First you must enter the value for the x-plane since the y-plane value follows the x-plane value. With the gaussian algorithm these parameters can be used to determine the maximum beam quality value M^2 possible within the defined apertures, see next section. With the wave optics code the apertures are used to take into account diffraction effects. If the x- and the y-plane values are different the BPM code uses an elliptical aperture. The default value for the radii is 1000000 μm , which is much greater than a usual beam spot size. To create an element that acts as an aperture only, use a dielectric interface with very large radius of curvature.

6.1.4 Tab "Miscellaneous"

Use the box "Wave Length, μm " to change the wavelength.

The box "Pump power for rescaling, W" you can be used to investigate dependence of the thermal lensing effect on the pump power. After changing the entry and clicking "Apply" or pressing the RETURN key the parabolic coefficients resulting from the parabolic fit of the FEA data (see section 6.13) are linearly rescaled corresponding to the ratio between original pump power and the value entered for rescaling (see section 6.14).

The shadowed box labeled with "Pump power" shows the value originally used for the FEA, it is for information only.

The beam quality parameters M_x^2 and M_y^2 for the x- and y-plane can be entered directly, or a higher order Hermite-Gaussian mode TEM_{0n} or TEM_{m0} can be selected which defines the beam quality automatically by the use of the equations $M_x^2 = 2n+1$ or $M_y^2 = 2m+1$, respectively. Alternatively you can define apertures and check one or both of the boxes "Use x(y)-plane apertures to define $M_{x(y)}^2$ " to compute the maximum M^2 possible within the defined apertures. In all cases the shape of the mode plot is being rescaled corresponding to the related highest order transverse mode.

In the case of a separate external beam, the parameters M_x^2 and M_y^2 must defined together with the other entrance plane parameters in the window "Starting Conditions of External Beam" as shown in section 6.7.

6.1.5 Tab "Spot Sizes"

The upper two rows of this index card show the gaussian spot sizes at the positions of the face elements for the x-plane. If non vanishing parabolic gain coefficients are defined the wave propagating from left to right differs from the wave propagating from right to left as explained in the Quick Guide.

Boxes belonging to the wave propagating from left to right are green, boxes belonging to the wave propagating from right to left are red, Row 3 and 4 show the corresponding y-plane values.

To show spot sizes at intermediate positions refer to section 6.6. "**Additional Beam Parameters at Drag Bar Position**"

To print spot size results please use the menu items "Print to File/ ..." of the main LASCAD window.

6.1.6 Layout of the Window "*Parameter Field*"

The horizontal distance of the boxes in the window "Parameter Field" is chosen to fill the window appropriately. If not all boxes can be shown, you can increase the width of the window. For this purpose drag the right border with the mouse or use the maximize button in the task bar of the window. Instead, you can use the scroll bar at the bottom of the window. Click the arrows of this bar to scroll the element numbers in single steps, or move the slider to do larger steps. You also can click into the fields left and right of the slider.

The parameter field window can be minimized; in this case element parameters can be modified by the use of the "**Modify Parameters**" dialog as explained in section 6.2.1.

6.2 Window with Gaussian Mode Plot

The caption of this window shows the type of resonator as being selected for example "Standing Wave Resonator". If a thermally lensing crystal is present in the cavity, additionally the FEA results directory is shown.

For rotationally symmetrical configurations, only one mode plot is shown. To get a nice graphical appearance the mode shape is visualized by five lines; the top line is related to the scale left of the plot.

For astigmatic configurations two mode plots are shown. The upper picture box shows the mode shape in the x-z-plane, the lower one the mode shape in the y-z-plane, respectively.

The numbers at the bottom of the lower picture box give the distances between the elements corresponding to the distance values shown in row three of the window "Parameter Field".

If parabolic gain parameters are defined in the row "Gain Param." in the window "Parameter Field", the plots may show green and red lines corresponding to the waves traveling from left to right and from right to left, respectively, as described in section 6.1.5.

Elements are represented by vertical lines. To characterize the type of the element, triangular symbols are used, which correspond to the element type selected in the window "Parameter Field". The following symbols are used:

- **mirrors:** filled rectangular triangles,
- **dielectric interfaces:** open rectangular triangles,
- **lenses:** filled isosceles triangles

Click the drop-down boxes in the row "Type-Parameter" of the parameter field, and confirm with the RETURN key to become familiar with the meaning of the symbols. The sign of the "Type-Parameter" is symbolized by the orientation of the triangles.

Below the picture boxes you see a **scroll bar** combined with a **Shrink & Stretch** control. With the scroll bar the diagrams can be moved to the left or to the right. The triangular buttons of the Shrink & Stretch control can be used to increase or decrease the width of the diagrams.

Check the box "**Adjust diameter**" at the bottom of the window to adjust the maximum spot size to the maximum height of the diagram.

Check the box "**Adjust Length**" box to adjust the length of the mode plot to the width of the picture box.

Below the mode plot a **slider** is shown which can be used as follows. When a box in the parameter field is clicked on, the related parameter, such as the curvature of a mirror is connected to this slider, and can be changed by moving the slider. The name of the parameter is specified in the leftmost box of the row containing the slider. Its value is shown in the yellow box to the right of the name. The third box to the right determines a percentage range within which the parameter can be modified. By default it is set to $\pm 50\%$ of the original parameter value, but another value can be entered. While moving the slider you can watch the actual value of the parameter in the yellow box. After releasing the mouse button the mode shape, corresponding to the changed parameter value, is plotted. Clicking into the parameter box or the percentage box centers the slider at the actual value of the parameter. The slider control cannot be connected to the boxes of the index card "**Apertures**".

At the lower right corner of the window you can see a group of three option buttons having the following function:

"Mode only": Only the mode shape within the resonator is computed and displayed.

"Mode + External Beam": The mode shape within the resonator is displayed together with the external beam. The external beam is characterized by a yellow color. Angle of incidence and refractive indices are taken into account when the beam is propagated through the right end mirror of the resonator.

"External beam only": Only the external beam is shown. The starting plane values for the external beam are identical to those used when "**Mode + External Beam**" is checked. To consider an independent external beam, start a new project and select the option "External Beam".

Click the menu item "**Show Additional Items/Stability Diagram**" to open the window "**Stability Diagram**". It is only available for standing wave resonators. For ring cavities a stability diagram cannot be defined.

Click the menu item "**Show Additional Items/Stability Criteria**" to open the window "**Stability Criteria**" which also is available for ring cavities.

Click the menu item "**Show Additional Items/ Additional Beam Parameters**" to open the window "**Additional Beam Parameters at Drag Bar Position**" as described in section 6.6. It is used in combination with the **magenta vertical bar** found at the right border of the picture box, which can be dragged over the mode plot diagrams. While doing so, its position with respect to element No. 0 is displayed in the magenta number box in the window "**Additional Beam Par. ...**". Simultaneously spot sizes and additional beam parameters are shown as described in section 6.6. Wait for the cross shaped mouse pointer before you begin to drag.

Click the menu item "**Show Additional Items/Gaussian Mode Profile**" to open the window "**Gaussian Mode Profile**", which again is used in combination with the magenta drag bar. It shows the mode profile at the position of the drag bar, as described in section 6.8.

6.2.1 Moving, inserting and clearing elements

These operations are carried through with of the mouse, or with the mouse in combination with the keys SHIFT or CTRL as follows:

(See also the item "Help/Use of Hotkeys and Mouse Buttons" in the main LASCAD menu)

Move an element: In order to move elements click on the vertical lines (not the central number boxes !!) in the mode plot boxes and drag them along the mode plot. While doing so you can see the number values for the distances to the left and the right being changed. After release of the mouse button the recalculated mode shape is plotted.

Modify an element: Elements can be modified by the use of the window "Parameter Field", but if this window is closed you can modify parameters in the following way. Click a line element with the right mouse button to show the "**Modify Parameters**" dialog. Its setup corresponds to the "Parameter Field" window and can be handled in the same way. You can even trigger the slider control from this window. Having this window open and clicking different line elements in the mode plot window with the right mouse button you can show the corresponding parameter values.

Insert an element: Press the **SHIFT** key and click with left mouse button into a field between two elements. The dialog "**Insert Elements**" appears showing the parameters of the element to be inserted. The setup of this window corresponds to the window "Parameter Field". For the sake of simplicity new elements always are inserted together with the definition of the properties of the medium at the left-hand side of the new element. Elements also can be added outside the right or left end mirror of the resonator. If the option button "Resonator + External Beam" is checked, and you are clicking into the field between right end mirror and the next external element to the right, additionally two option buttons appear asking you to decide, if the new element shall belong to the resonator or to the external beam.

Astigmatic elements can be defined by the use of the tabs "x-plane parameters" and "y-plane parameters". To clear astigmatism press the **CTRL** key, and click into the concerned number box which resets the y-plane value to the x-plane value. For non astigmatic parameters the y-plane value always follows the x-plane value.

Clear an element: Press the **CTRL** Key and click the line of the element to be cleared. A confirmation dialog is appearing.

Insert a thermally lensing crystal: Please refer to sections 6.10 and 6.13.

Clear a thermal lens: To clear a thermal lens press the **CTRL** button and click with the left mouse button into the field of the thermal lens. After clearing a thermal lens its end faces remain in the cavity and must be cleared separately.

6.3 Main Window "LASCAD"

6.3.1 Drop-Down Menus

6.3.1.1 "File"

"**New Project**" opens the Window "*New Project*" which is described in section 6.4.

"**Open Project**" reads all parameters including thermal lens parameters of a previous project from a file to which they have been saved before.

"**Save Project**" and "**Save Project As**" saves all parameter values with explanations in ASCII format to a file. You can edit and print this file with a text editor, but be careful in changing values, never clear lines.

"**Exit**" closes LASCAD.

6.3.1.2 "Print"

"**Mode Plot**" opens the printer dialog and prints the gaussian mode shape shown in the mode plot window.

"**Parameter Field**" prints the parameter field shown in the window "Parameter Field" to a printer. To get it on a printer page it may be advisable to reduce the horizontal extension of the window in advance or to print it in landscape format.

"**Parabolic Fit**" prints the graphs of the window "Parabolic fit of Temperature and Deformation ...", see section 6.13.

6.3.1.3 "Print to File"

"Mode Plot" generates a bitmap file of the mode plot diagrams.

"Optical Element Parameters" generates a file containing a list of the optical parameters shown in the window "Parameter Field". The elements are listed with increasing number together with their position along the optical axis, element type, curvature or focal length, and the parameters of the medium between the elements.

"Spot Sizes along Resonator axis" generates a file where the spot sizes of the gaussian mode inside the cavity are listed in small steps with increasing z-coordinate. The list contains the spot sizes of the wave traveling from left to right as well as of the wave traveling from right to left, also y-plane spot sizes are listed if applicable.

"Spot Sizes along External Beam" generates a file where the spot sizes of the external gaussian beam are listed in small steps with increasing z-coordinate.

"Spot Sizes at Element Positions" generates a file where the spot sizes are listed at the element positions together with the element type.

"Intensity at Right Mirror" generates a bitmap file of the intensity distribution at the right mirror as computed by the BPM code, see section 8.4.

"Phase at Right Mirror" generates a bitmap file of the phase distribution at the right mirror as computed by the BPM code, see section 8.4.

"Eigenmode" generates a bitmap file of the eigenmode profile shown in the window "Eigenmodes", see section 8.6.

6.3.1.4 "Copy to Clipboard"

"Copy to Clipboard" copies bitmaps of the items described in the previous section to the clipboard.

6.3.1.5 "View"

"Gaussian Mode Plot" reopens the gaussian mode plot window.

"Parameter Field" reopens the window "Parameter Field".

"Mode Profile" opens the window "Mode Profile" as described in section 6.8.

"Stability Diagram" opens a window showing a stability diagram, see section 6.5.

"Stability Criteria" opens a window showing values for resonator stability criteria, see section 6.5.

"Crystal, Pump Beam, and Material Parameters" opens or reopens a window to enter the corresponding parameters, see section 6.10.

"Parabolic Fit" opens or reopens a window to start the parabolic fit computation see section 6.13.

"Input for External Beam" opens or reopens the window "Entrance Plane Beam Parameters" to enter the beam parameters of an external beam at the starting plane. This button can only be used if the **"External Beam"** option of the **"New Project"** window was checked or if a project with this option was opened.

"Pump Profile" opens or reopens the window showing the Pump beam profile, see section 6.12.

"Curvature of Phase Front" opens the window **"Curvature of Phase Front"**, which shows a plot of the phase front curvature, see section 6.9.

6.3.1.6 "FEA"

"Parameter Input & Start of FEA Code" opens a window to select a cavity configuration, and to enter crystal dimensions, pump configuration, and material parameters, and to start the FEA code as described section 6.10.

"2D Data Profiles and Parabolic Fit" opens a window to start the parabolic fit computation using the file FEA.Out in the working directory which contains the results of the FEA carried through previously together with all crystal, pump and material parameters used with this computation, see section 6.13.

"3D Visualizer" opens a window to show the FEA results as described in section 6.11.

6.3.1.7 "CW Laser Power"

Opens a window to compute the laser power output for cw operation.

6.3.1.8 "Dynamic Multimode Analysis"

Opens a window to activate the code for dynamic analysis of multimode and Q-switch operation (DMA).

6.3.1.9 "BPM"

"Run BPM" opens a window to enter parameters for the BPM computation and to start the latter one as described in section 8.1.

"Show Beam Radius and Power Output" opens the window "Beam Radius and Laser Power versus Cavity iteration". It shows, how beam radius and laser power output develop with increasing number of cavity iterations, see section 8.2.

"Show Beam Quality" opens the window "Beam Quality versus Cavity iteration". It shows, how the beam quality parameters M_x^2 and M_y^2 develop with increasing number of cavity iterations, see section 8.3.

"Show Beam Profile" opens the window "Intensity and Phase at Right End Mirror", see section 8.4.

"Show Frequency Spectrum" opens the window "Spectrum of Eigenfrequencies", see section 8.5.

"Show Eigenmodes" opens the window "Eigenmodes", see section 8.6.

6.3.1.10 Tool Bar

From left to right:

1. Button: Opens the New Project Window (same function as corresponding menu item).
2. Button: Saves all parameters of the project to a file (same function as corresponding menu item).
3. Button: Opens a previous project file (same function as corresponding menu item).
4. Button: Prints the mode shape shown in the mode plot window to a printer (same function as corresponding menu item).

6.4 Window "New Project"

After you have pressed the button "New Project" in the tool bar the window "New Project" appears. Here you have the possibility to check one of the 3 options "Standing Wave Resonator", "Ring Resonator" or "External Beam" which are explained below. In addition you can enter the wave length [μm], the number of face elements, and values for the beam quality parameter.

6.4.1 Option: "Standing Wave Resonator"

This option opens a standing wave resonator with a proposal for the parameter values as described in section 5 "Starting the Program". Parameters can be changed in the window "Parameter Field". Additional elements can be inserted into the cavity.

6.4.2 Option: "Ring resonator"

This option opens a ring resonator. In a cavity of this type the null order element only serves as a dummy interface doubling the last element at the right hand side. It is therefore not possible to add elements outside the left end of the resonator. However, a physically equivalent result can be achieved by insertion of an element outside the last element or between null and first order element. When you enter the number of elements of a ring resonator, the null order element has not to be taken into account.

6.4.3 Option: "External Beam"

This option allows separate computation of an external beam. After checking this option and clicking OK, the window "**Starting Conditions of External Beam**" is opened as described in section 6.7. When you enter the number of elements for an external beam, the null order element has not to be taken into account.

6.5 Windows: "Stability Diagram" and "Stability Criteria"

In case of a standing wave resonator the check box "Show Stability Diagram" in the mode plot window opens the window "*Stability Diagram*", in case of a ring resonator the same check box now with name "Show Stability Criteria" opens the window "*Stability Criteria*". Both windows also can be opened by the use of the main menu items "View/Stability Diagram" or "View/Stability Criteria", respectively. The

window "Stability Criterions" additionally can be opened from the window "Stability Diagram".

As described in the book *LASERS*, Chapt. 19 of *A.E.Siegman*, the stability diagram initially has been developed for the stable two-mirror resonator. For this resonator it can be shown that real and finite solutions of the gaussian beam parameters and spot sizes only exist, if the relation

$$0 \leq g_1 g_2 \leq 1$$

is met, where g_1 and g_2 are defined by

$$g_1 \equiv 1 - \frac{L}{R_1} \quad \text{and} \quad g_2 \equiv 1 - \frac{L}{R_2} .$$

Here L is the distance of the mirrors and R_1 and R_2 are the radii of curvature of left and right mirror, respectively. As shown in *PRINCIPLES OF LASERS*, Sect. 5.5 by O. Svelto this concept can be generalized to the case of the general standing wave resonator with internal optical elements. In this case g_1 and g_2 must be replaced by the elements $A_1 = g_1^*$ and $D_1 = g_2^*$ of the one-way ABCD matrix, which is defined as the matrix product of all internal elements with the end mirrors being replaced by a combination of an equivalent internal lens plus a plane mirror. For this general case an equivalent stability condition

$$0 \leq g_1^* g_2^* \leq 1$$

can be deduced. If gain guiding is present these relations are not exactly valid, since in this case the one-way matrix depends on the propagation direction. However, for small parabolic gain coefficients still an useful approximation is obtained (see below).

In the "Stability Diagram" the point belonging to the actual configuration can be shown by pressing "Plot point ...". If you are changing now the configuration and plot a new point, the old point is kept to show the difference. For instance you can see the effect of rescaling the pump power as follows. Open the tab "Miscellaneous" in the window "Parameter Field", enter a desired value into the related box, click "Apply", and show the resulting point in the "Stability Diagram". Even a new FEA can be carried through, but in this case the results files in the folder "FEA" are overwritten. To keep them the folder must be renamed in advance. To restore a configuration belonging to a previous point click into this point and follow the instructions in the window that pops up. To clear all points press "Clear points". Project files belonging to the individual points are stored in the subdirectory *Stabilityconfigs* of the working directory with names *Stabfile_1*, *Stabfile_2*, ... To additionally show the diagram for the y-plane mode check "Show y-plane diagram".

In case of a ring resonator the stability diagram cannot be used anymore, instead more general criterions like real and imaginary part of half the trace of the full round trip ABCD matrix, or the perturbation eigenvalues must be used as defined in *Siegman*, *Lasers*, Chapt. 21. The numerical values of these criterions along with those of

A_1 and D_1 are shown in the window "*Stability Criteria*". For a stable resonator the real part m of half the trace of the full round trip matrix should meet the condition

$$-1 \leq m \leq 1 .$$

The perturbation eigenvalue delivers a criterion for the stability of an eigensolution against perturbation; its absolute square value must be ≤ 1 .

To check applicability of the stability diagram in case of gain guiding, additionally the quantities $A_1 \cdot D_1$ and $(A+D)/4+1/2$, which are identical for vanishing gain, are shown in the window "*Stability Criteria*". Here A and D are elements of the full round trip ABCD matrix.

6.6 Window "Beam Parameters at Drag Bar Position"

To open this window click the item "**Show Additional Items/Beam Parameters**" in the menu of the mode plot window.

Drag the **vertical magenta bar** from the right edge of the picture box over the mode plot. The distance of the bar from element No. 0 is shown in the magenta box. Alternatively, you can enter a value into this box to change the position of the bar. If the bar is lost outside the picture press the button "**Reset Drag Bar**". The number boxes below show the

- spot size at the position of the drag bar,
- radius of the phase front curvature at the position of the drag bar,
- far-field beam angle,
- spot size (= radius) at virtual beam waist,
- position of virtual beam waist,
- beam parameter product

The virtual beam waist is meaningful for sections along the cavity with no parabolic refractive index and gain. For those sections the virtual waist is the waist of an equivalent gaussian beam with identical local spot size and phase front curvature which however can propagate undisturbed by the optical elements in the cavity. If a real beam waist is present in the section being considered the virtual waist is identical with this real waist. In sections with non-vanishing parabolic refractive index or gain parameter the virtual beam waist may not coincide exactly with the real beam waist.

The beam parameter product is the product (radius at waist * beam divergence) as explained in the book "Laser Resonators and Beam Propagation" of N. Hodgson and H. Weber, Springer, Berlin 2005.

Green and red boxes distinguish between the wave propagating from left to right and from right to left, respectively, as explained in section 6.1.5.

To print spot size results use items "Print to File/ ..." in the LASCAD main menu.

6.7 Window "Starting Conditions of External Beam"

This window is opened automatically after you have checked the option "**External Beam**" in the window "**New Project**". Additionally, it is possible to reopen this window

by clicking the item "**Starting Conditions of External Beam**" in the menu of the mode plot window. You have three options to define the starting conditions of a beam:

- Spot size at starting point and far-field beam angle.
- Spot size at waist and the distance between starting point and beam waist.
- Far-field beam angle and the distance between starting point and beam waist.

All input data must take into account the refractive index of the medium where the beam is propagating.

The distance between starting point and beam waist should be defined as an absolute value. To toggle between con- and divergent beam, click into the box with sign ">" or "<".

In addition, the beam quality parameters M_x^2 and M_y^2 for x-z- and y-z-plane can be defined. In case of a multimode beam the spot size is the radius of the highest order transverse mode T_{n0} which can be approximately related to beam quality by the relation $M_x^2 = 2n+1$. In the case of a separate external beam the beam quality parameters cannot be modified in the tab "Miscellaneous" of the window "Parameter Field", since they are related to the other starting conditions.

All parameters can be defined independently for x-z- and y-z-plane. After defining a y-z-plane parameter different from the corresponding x-z-plane parameter, the background of the corresponding box becomes blue indicating, that astigmatism is set for this parameter. If astigmatism is set, the corresponding x-z-plane value can be changed independently of the y-z-value, otherwise the input for y-z-plane follows the input for x-z-plane.

Press the CTRL key and click into a box with blue background to set the y-z-plane parameter back to the x-z-plane parameter.

6.8 Window "Gaussian Mode Profile"

This window can be opened by clicking the menu item "**Show Additional Items/ Gaussian Mode Profile**" in the mode plot window as already described in section 6.2.

Move the mouse pointer into the mode plot window, and drag the magenta vertical bar from the right end over the mode plot to show the Hermite-Gaussian modes TEM_{n0} and TEM_{0m} along the x- and y-axis for the position of the magenta bar.

Transverse modes of different order can be shown by the use of up-down arrow controls in the lower right corner of the window.

Use the spin buttons to change vertical and horizontal scale, or alternatively, type directly into the corresponding boxes, and click the "**Apply**" button. A mouse click into one of the graphs fits the height of the graph to the diagrams height.

The number box at the lower left corner shows the distance of the magenta drag bar from element number zero in the mode plot window.

If you have a thermal lens in your mode plot, you can additionally display the transverse pump profile. Check "**Show pump light distribution**" for this purpose. The

pump profile is shown in W/mm^3 as the right ordinate indicates. The scale of this ordinate is self-adjusting, and currently, cannot be changed by the user.

The "Print" button allows you to print the diagrams.

This window also can be opened by clicking of LASCAD main menu item "View/ Mode Profile".

6.9 Window "Curvature of Phase Front"

Clicking "View/Curvature of Phase Front" in the menu bar of the main LASCAD window opens the window "Curvature of Phase Front" that shows the phase front curvature of the gaussian mode. This is especially interesting if gain parameters are present, since a parabolic gain parameter causes an additional curvature. Generate a configuration with two plane mirrors where only a parabolic gain parameter is present, then you can see, that the wave traveling from left to right has a positive curvature to the right hand side whereas the wave traveling in the opposite direction has a curvature to the left hand side i.e. the curvature is always positive in the propagation direction. On the other hand if you consider a configuration where only the parabolic refractive index is different from zero the phase front is plane. You can adjust the scale used for displaying the curvature by the use of the slider. The left most position corresponds to the real curvature.

6.10 Window "Crystal, Pump Beam, and Material Parameters"

Click "FEA/Parameter Input & Start of FEA Code" in the menu bar of the main LASCAD window to open the window "*Crystal, Pump Beam, and Material Parameters*". This window is showing 6 tabs which are described in the following.

6.10.1 Tab "Models"

This tab is offering 8 options to select crystal geometry and pumping layout:

- Single or dual end pumped cylindrical rod
- Cylindrical rod with top hat shape of pump light along z-axis and supergaussian shape perpendicular to z-axis
- Single or dual end pumped slab
- Slab with top hat shape of pump light along z-axis and supergaussian shape perpendicular to z-axis
- Side pumped cylindrical rod
- Side pumped slab including sandwich configurations
- Cylindrical rod with numerical input of pump light distribution
- Slab with numerical input of pump light distribution

Below the frame containing the option buttons, boxes are found to define length, diameter, or width and height of the crystal.

For slabs the front faces can be tilted. In this case the length of the crystal is the distance between the centers of the two front faces. The front faces are numbered in z-direction which is parallel to the long dimension of the slab. The origin of the coordinate system is located in the centre of the left front face. The faces are rotated about an axis parallel to the y-axis. The angles represent the angle between face and positive z-axis. 90°

means that the face is at right angle with z-axis. A negative value means that the angle between z-axis and face is greater than 90° .

When the FEA is completed the "3D Visualizer" can be used to show the origin of the coordinate system, as described in section 6.11.

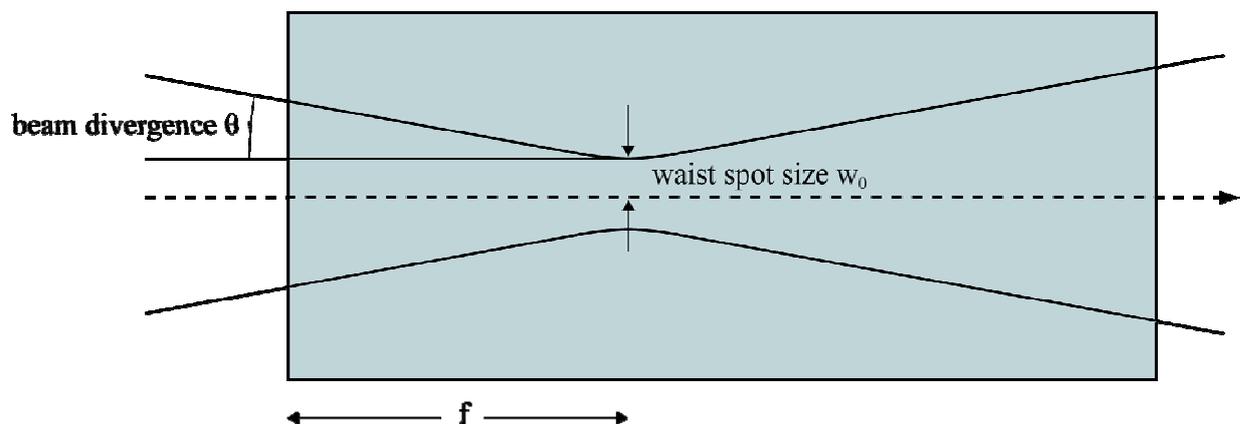
6.10.2 Tab "Pump Light"

LASCAD is computing the absorbed pump power density by the use of analytical approximations involving super-gaussian functions as described in sections 6.10.2.1 to 6.10.2.4. There are however interfaces available to the ray tracing codes TracePro and ZEMAX. With these commercial ray tracing codes the absorbed pump power density can be computed numerically. The 3D data set generated with these programs can be used as input for the FEA as described in sections 6.10.2.5 and 6.10.2.6.

The content of the index card "Pump Light" depends on the model being selected.

6.10.2.1 End Pumped Rod or Slab

In case of end pumping the intensity distribution of the pump beam i.e. the absorbed power density Q [W/mm^3] is modeled by the use of a super-gaussian function. The pump beam is assumed to be focused into the crystal as shown in the picture. The pump beam can enter the crystal from the left or from the right end, or from both ends.



Pump beam focused into the crystal, f is the distance of the waist from the left end face

Use the boxes in the frame "Incident power" to define the total power of each beam individually, and be sure to set the power of a beam not being used to zero. After clicking into one of the boxes for the incident power the parameter boxes above automatically are rearranged in order to allow for inputs for the left or the right beam, respectively.

Option buttons allow selecting rotational or x-y-symmetry.

The following Eqs. describe the local density of the absorbed pump power of the left (L) and the right (R) beam, respectively.

For **rotational** symmetry the distribution is defined by

$$Q_{L(R)} = \frac{\alpha \beta P_{L(R)}}{C \pi w^2} \exp \left[-2 \left| \frac{r}{w} \right|^{SG} - \alpha z_{dop,L(R)} \right], \quad (1)$$

with

$$r = \sqrt{(x - \Delta x)^2 + (y - \Delta y)^2}, \quad (2)$$

$$w = \sqrt{\left[w_{0,L(R)}^2 + \left((z_{L(R)} - f_{L(R)}) \theta_{L(R)} \right)^2 \right]}, \quad (3)$$

$$C = 2^{-2/SG_{L,R}} \Gamma(1 + 2/SG_{L,R}), \quad (4)$$

Here is $P_{L(R)}$ [W] is the total pump power of the left or the right beam, respectively. Γ is the Gamma function. for $SG=2$ Eq. (4) delivers $C=1/2$.

α [1/mm] is the absorption coefficient and β the heat efficiency coefficient that defines the relative part of the absorbed pump power converted into heat (fractional thermal load). α and β can be entered into the index card "*Material parameters*" described below. For Nd:YAG for instance β has a value of ~ 0.3 .

$z_{L(R)}$ is the distance from the centre of the left face or the right face of the crystal, respectively.

$z_{dop L(R)}$ is the distance from the left or the right end of the doped (or pumped) region, respectively.

$\theta_{L,R}$ is the far-field half angle of the pump beam in the crystal (angle where the intensity drops to $1/e^2$). It has to be entered into the box labeled with "Beam divergence, mrad".

$f_{L(R)}$ defines the distance of the pump beam waist from the left or the right end of the crystal, respectively. Positive or negative values of $f_{L,R}$ mean that the waist is located inside or outside the crystal, respectively.

w_0 [μm] is the spot size of the pump beam waist. According to general use spot size is the radius where the intensity drops to $1/e^2$.

SG is the super-gaussian exponent. For $SG=2$ one obtains a common gaussian transverse profile. With increasing SG the profile approximates a top-hat distribution which is obtained for $SG \rightarrow \infty$.

By the use of the parameters Δx and Δy the influence of a lateral displacement of the axis each pump beam on mode formation can be studied. This lateral displacement is taken into account in the FEA and in the wave optics code, but not yet in the gaussian algorithm.

For **x-y-symmetry** the distribution is defined by

$$Q = \frac{\alpha \beta P}{C_x C_y w_x w_y} \exp \left[-2 \left| \frac{x - \Delta x}{w_x} \right|^{SGX} - 2 \left| \frac{y - \Delta y}{w_y} \right|^{SGY} - \alpha z_{dop} \right] \quad (5)$$

with

$$w_x = \sqrt{[w_{0x}^2 + ((z - f_x) \theta_x)^2]} \quad (6a)$$

$$w_y = \sqrt{[w_{0y}^2 + ((z - f_y) \theta_y)^2]} \quad (6b)$$

and

$$C_x = 2^{1-1/SGX} \Gamma(1 + 1/SGX) \quad (7a)$$

$$C_y = 2^{1-1/SGY} \Gamma(1 + 1/SGY) \quad (7b)$$

Here θ_x and θ_y are the far-field pump beam half angles in x-z- and y-z-plane. f_x and f_y define the distances of the pump beam waist from the entrance plane, w_{0x} and w_{0y} the x- and y- component of the spot size of the pump beam waist. The parameters in Eqs. (5) to (7) have to be interpreted for each beam correspondingly to Eqs. (1) to (4).

Again all parameters except α and β can be defined for the beams coming from left and right individually though not indicated in Eqs. (5) to (7) explicitly.

Press the button "**Show Pump Profile**" to open the window "**Pump Profile**" that visualizes the absorbed pump power density as described in section 6.12.

6.10.2.2 Pump Light Distribution with Top Hat Profile Along Crystal Axis

This type of input is providing an approximate model for cases where it is not possible to simulate the absorption of the pump light in detail analytically. The user must know the pump light distribution approximately, e.g. for flash lamp pumping or for side pumping. It is assumed that the pump light distribution is constant over a certain region along the crystal axis (z-direction) with limits that can be defined in the index card "Doping & Materials". The transversal distribution again is described by the use of super-gaussian functions with options for rotational and x-y-symmetry. For this type of pump light distribution the normalization is carried through by the FEA code. The pump power entered into the box "Total pump power" therefore must correspond to the integral over the pump power density absorbed in whole the crystal volume.

Rotational symmetry

$$Q = \frac{\beta P}{C \pi w^2 L_p} \exp \left[-2 \left| \frac{r}{w} \right|^{SG} \right] \quad (8)$$

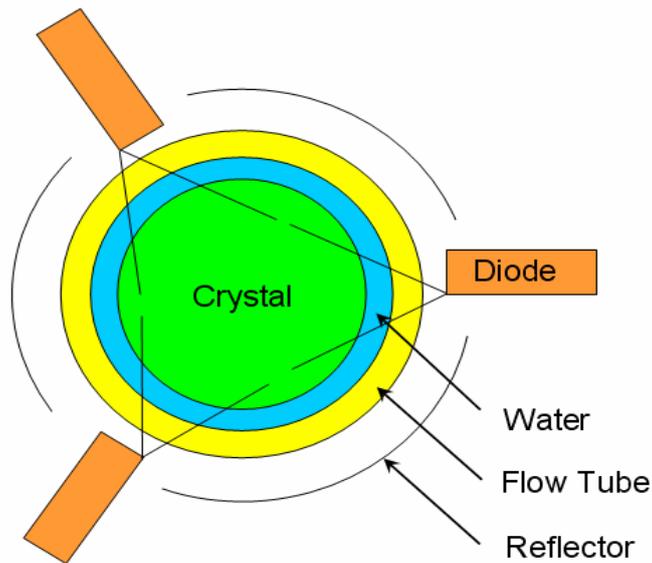
Here L_p is the length of the pump light distribution in z-direction. Click the tab "Doping & Materials" to define where the pumped region begins and where it ends along the z-axis. With the default entries the whole length of the crystal is used for L_p . The other parameters are defined by Eqs. (2 – 4).

X-Y-Symmetry

$$Q = \frac{\beta P}{C_x C_y w_x w_y L_p} \exp \left[-2 \left| \frac{x - \Delta x}{w_x} \right|^{SGX} - 2 \left| \frac{y - \Delta y}{w_y} \right|^{SGY} \right] \quad (9)$$

Here L_p again is the length of the pump pumped region along the z-direction as described above. The other parameters are defined by Eqs. (6 and 7).

6.10.2.3 Side Pumped Rod



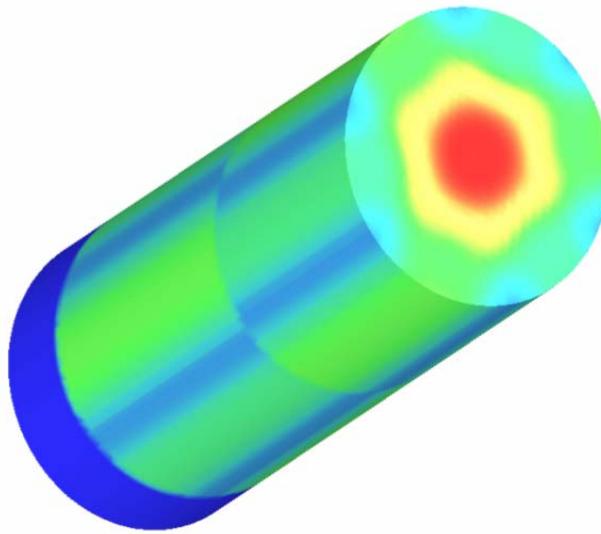
Most entries for the tab "Pump Light" are self-explanatory in case of a side pumped rod. It is assumed that a flow tube surrounds a cylindrical rod with a cooling fluid between tube and rod as shown in the picture. Outside the flow tube a reflector is set up. Its distance from the crystal axis and its radius can be defined by different entries. Therefore, the reflector must not surround the crystal concentrically.

The length of the cooled section can be defined in the tab "Boundaries".

"Total pump power" is the pump power added up over all diode bars.

"Length of diode bars" is the length of the slow axis of a single bar. Divergence with respect to the slow axis is not considered, the length therefore has to be defined to include this divergence approximately.

If more than one group of diode bars are positioned along the crystal, the groups are automatically offset with respect to each other by an angle of $360^\circ/2n$ where n is the number of bars around the rod. The picture is showing the angular offset of pump light distribution for an irradiation with diodes arranged as triplets.



If you have no offset enter "1" for "Number of diodes in axis direction of crystal" and multiply the entry for "length of the diode bars" with the number of diode bars along axis.

The entry "x-coordinate of pump beam intersection point" allows to move the intersection point of the irradiation directions away from the crystal axis for instance to compensate unsymmetrical heating in case 2 diode bars irradiating under 90° .

It is assumed that the beam coming from the diode is propagating like a gaussian beam in a plane perpendicular to the slow axis. The FWHM of the diodes is used to compute the gaussian far-field angle in this plane which is defined as the angle where the intensity drops to $1/e^2$. For the computation a super-gaussian intensity distribution with exponent 4 with respect to the fast axis is used. Input of the super-gaussian exponent from the GUI will be provided shortly.

Press the button "**Show Pump Beam**" to see the plot of the shape of a single pump beam in a plane perpendicular to the crystal axis. It shows the pump beam as it propagates from the diode through the flow tube, the crystal, and again through the flow tube. After reflection at element 7 it propagates once more through flow tube and crystal. The computation of this plot takes into account all curvatures and refractive indices, of crystal, flow tube and reflector. Since the gaussian algorithm takes into account the coherence of the pump beam it may provide better results than a ray tracing code until the first reflection. After the first reflection aberrations are expected to become important, and therefore, it may be that this method only delivers approximate results for the reflected part of the beam. By this reason, an algorithm has

been installed that broadens the pump beam slightly if its diameter becomes smaller than $40\ \mu\text{m}$. Comparison with ray tracing results has shown agreement up to a few percent.

In the plot of the shape of a single pump beam a second reflection is indicated at element 11 corresponding to the barrel surface of the rod, after the beam has passed the rod on the way back. This second reflection is not taken into account for the computation of the pump light distribution, but can be included on request.

The computational method described above also can be used for cases where the rod is directly illuminated by the diode without a flow tube in between. In this case the thickness of the flow tube should be made small and the refractive indices of flow tube and cooling fluid must be set equal to the cooling medium between diode and rod.

Use the button "**Show Pump Light Distribution**" to show the absorbed pump power density along x and y axis at different cross sections along the z-axis.

6.10.2.4 Side Pumped Slab

This configuration concerns a slab with rectangular cross section that is pumped from one side or symmetrically from two opposite sides with diode bars whose slow axis is oriented parallel to the longitudinal axis of the slab. In the latter case check the box "**Symmetrically pumped from two opposite directions**". In the following it is assumed that this box is not checked.

The z-axis of the coordinate system is parallel to the longitudinal axis of the slab and goes through the centre of the rectangular cross section. The x-axis is parallel to the width of the slab, the y-axis parallel to the height. When the FEA computation is completed you can press the button with the "wire frame slab" icon in the menu of the window "3D Visualizer" and click into the plot, to see the triade representing the origin (see section 6.11).

The distance of the diode bar from the z-axis is equal to $0.5 \cdot \text{width}$ plus the distance of diode from the plane through which the beam enters into the crystal. This latter distance is assumed to have a negative value to be consistent with end pumping. The pump light impinges the slab in positive x-direction.

In the column "**x-z-plane**", the entry "**Spot size**" is $0.5 \cdot \text{length}$ of diode in slow axis direction. If several diodes are arranged along a straight line, it is the length of all diodes together. In z direction the diodes are assumed to be arranged symmetrically with respect to the centre of the crystal. To take into account an asymmetric position a non vanishing value can be entered into the box "Displacement ..."

In the column "**x-y-plane**", the entry "**Spot size**" is the spot size of the diode at the exit plane of the diode in fast axis direction. According to general use, the spot size always is the radius where the intensity drops to $1/e^2$. The x-y-plane spot size should be small for instance $1\ \mu\text{m}$, but the latter is not very important, since the pumped region is far outside the so-called Rayleigh region and therefore the beam expands linearly. Also in y direction the diodes are assumed to be arranged symmetrically with respect to the

centre of the crystal. To take into account an asymmetric position a non vanishing value can be entered into the box "Displacement ..."

In the column "x-y-plane", the entry "**Beam divergence**" is the divergence of the diode beam in fast axis direction i.e. the angle where the intensity drops to $1/e^2$. For a gaussian distribution the relation between the FWHM (degrees) and the divergence (mrad) is given by $\text{divergence} \sim 0.015 * \text{FWHM}$ where the factor corresponds to $\pi/360 * \sqrt{2/(-\ln(0.5))}$. For a supergaussian distribution the divergence is closer to $\text{FWHM}/2$, if both are given in the same units. The beam divergence in slow axis direction (column "z-plane") usually is small.

"**Super gaussian exponent**" exponent describes the shape of the pump beam. Along z-axis (slow axis) it should be a big number for instance 500 or 1000000 to define almost rectangular shape. You can see the profile by pressing the button "Apply + Show Pump Beam Profile", see section 6.12.

"**Refractive index of slab for pump wave length**" must be defined to take into account the refraction of the pump beam when it enters the crystal. In case of 2 materials (see below) it is the refractive index of the laser material.

The extension of the doped region along the x-axis can be restricted to values $> 0.5 * \text{width}$ and $< 0.5 * \text{width}$, see register card "Doping".

In case of sandwiched slabs different materials can be assumed for doped and undoped region.

6.10.2.5 Cylindrical Rod with Numerical Input of Pump Light Distribution

This configuration concerns a cylindrical rod with the absorbed pump power density being computed numerically. The distribution of the absorbed pump power density can be computed by the use of the program TracePro which is a product of Lambda Research Corporation (<http://www.lambdares.com>), or by the use of ZEMAX which is a product of ZEMAX Development Corporation (<http://www.zemax.com>). Data generated with either of these two programs can be used as input for LASCAD which interpolates the 3D data set with respect to the grid used with the FEA code.

If the tab "Pump Light" of the window "Crystal, Pump Beam, and Material Parameters" is checked two frames are shown. The upper frame shows two option buttons to select input from TracePro or from ZEMAX. The lower frame offers two options concerning the normalization of the input data.

If "**Integrate input data over crystal volume, and show result in the box for totally absorbed pump power**" is checked, the input data are integrated over the crystal volume, but are not normalized. The result of integration is shown in the box "**Totally absorbed pump power, W**" when FEA is completed.

If "**Normalize input data, and multiply them with totally absorbed pump power as entered above**" is checked, the input data are normalized, and then are multiplied with the value entered in the box "**Totally absorbed pump power, W**"

If the input data set is generated by the use of TracePro it is important to define the origin of the coordinate systems identically in TracePro and in LASCAD.

If ZEMAX is used the coordinate origin of the input data always must be the geometrical centre of the crystal that means that the latter one must be positioned in the centre of the detector volume used with ZEMAX.

Entries for the tab "Doping & Materials" have no meaning with this configuration.

After pressing the button "Apply and Run FEA" a dialog appears that allows opening the file with the input data. It is recommended to copy this file in advance to the LASCAD working directory or to the subdirectory FEA of the latter one.

Cooling boundaries and material properties can be defined in the same way as for the other configurations.

Information how to generate the input data set with TracePro can be found in the files TracePro Volume Flux Calculations.doc (or *.pdf) in the Directory Documentation on the CD-ROM. In case of ZEMAX the user is related to the ZEMAX manual.

6.10.2.6 Slab with Numerical Input of Pump Light Distribution

This configuration concerns a slab with absorbed pump power density being computed numerically. It can be handled in the same way as the cylindrical rod with numerically computed pump light distribution as described in sect. 6.10.2.5. Especially in case of slabs with tilted front faces correct positioning of the origin of the input data is important. Concerning the definition of the origin used with LASCAD see section 6.10.1.

6.10.3 Tab "Boundaries"

The entries for this tab are defining cooling boundaries. The layout of the tab depends on crystal geometry and pump configuration. The modern semi-structured meshing technology used with the FEA code allows very flexible adjustment of the cooling boundaries to different experimental requirements.

Rods can be cooled at barrel surface, and at front and back face at circular areas around the axis. Slabs can be cooled at all four lateral surfaces. The size of the cooled area can be defined individually by the use of the entries "**Surface extends from z = ... to z = ...**", for this purpose the correct position of the origin as described in section 6.10.1 must be taken account.

Also the temperatures of the surfaces can be defined individually. To define temperatures the Kelvin scale has to be used, however the zero point can be set arbitrarily as long as temperature independent material parameters are used.

To select fluid i.e. convective cooling the corresponding boxes must be checked. In the latter case Newtons law of cooling

$$\vec{q} = h_f (T_s - T_B) \quad (10)$$

is used. Here T_B is the bulk temperature of the adjacent fluid, h_f is the Film coefficient, and T_s is the surface temperature of the crystal. \vec{q} is the heat flux perpendicular to the surface into the adjacent medium. In this case the values entered for the cooling temperature are interpreted as T_B .

The "**Reference temperature**" is used with computation of thermal deformation and corresponds to the crystal temperature before heating. It is very important to define the reference temperature consistent with the temperatures of the surfaces. For instance, if the surface temperatures are defined using the Celsius scale and the reference temperature is defined using the Kelvin scale completely wrong results are obtained for the thermal deformation.

If temperature dependence of material parameters is taken into account - see next section, the unit "Kelvin" must be used to define the boundary and the reference temperatures, otherwise the temperature zero point is not important.

6.10.4 Tab "Material parameters"

The parameter values shown for material A after starting a new project are the values for Nd:YAG. If you change values, always begin with the x-component, since the other components follow the x-component. Coefficients for thermal conductivity, thermal expansion, refractive index, and dn/dT can be defined individually for each coordinate axis, for other parameters orthotropic (= axis dependent) use of material properties is under development.

Concerning definition and computation of the **absorption coefficient** please refer to the Appendix. For some laser materials the absorption coefficient depends on the polarization of the pump light, for instance Nd:YVO₄ as shown in Fig.2 in the Appendix. This can be taken into account for end pumped configurations, for which different values of the absorption coefficient can be defined for two polarization directions perpendicular to each other. The percentage of the total pump power, which is polarized in || direction, can be entered into the box "**Percentage of pump power ||**".

The **heat efficiency factor** defines the relative part of absorbed pump power converted into heat also referred to as fractional thermal load.

The parameters **stimulated emission cross section**, **spontaneous fluorescence lifetime** and **pump efficiency** are needed for the computation of laser power output, and are explained in sect. 6.14.

Check the box "**3-Level-Laser-System**" to take into account rate equations for 3-level-systems with the computation of laser power output as described in sect. 6.14.

Press the button "**Show Material Parameters for 3-Level-Systems**" to open the window "*Parameters for Quasi-3-Level Lasers*" as described in section 6.15.

The 3 option buttons following below allow for selecting temperature dependence of parameters and to involve a second material B. By default, the option button "**One material with temperature independent parameters**" is checked.

To take into account temperature dependence for material A check the button "**One material with temperature dependent parameters**". The code uses rough approximations of temperature dependence for YAG material which on request can be replaced by user defined equations for temperature dependency of any material. If you are taking into account temperature dependence, you must use the **Kelvin Scale (!)** to define the boundary and the reference temperature in the tab "Boundaries".

To take into account two different materials the option button "**Two materials A and B where material B can be a heat sink material**" has to be checked. Also in this case parameters of material A (and on request B) can be made temperature dependent by inclusion of user defined equations.

The box "**The beam is passing through material B**" has to be checked, if the laser beam propagates through material B in order to take into account the values for refractive index and dn/dT for material B.

In the subdirectory MATERIAL DATA FILES of the LASCAD application directory material properties of some frequently used laser materials can be found. To open these material files press the button "**Open material File**" at the lower border of the tab, alternatively the material files can be opened by a text editor.

You can save the actual material parameters into a material file with ASCII format. The box at the right hand side of the label "**Material A:**" allows for defining a material name that is written into the first row of the material file.

6.10.5 Tab "Doping & Materials"

The entries for this tab depend on the option selected in tab "Material Parameters".

If one of the two option buttons with caption "**One material ...**" is checked, only doped and undoped regions of the crystal can be defined by the use of the boxes below the label "**Pumped region extends**".

By default the input boxes are showing very big numbers defining a big volume extending from the x-y-plane in positive z-direction. Therefore, if you do not change anything the whole crystal is contained in this volume, and consequently is being pumped. To define for instance a crystal with undoped end caps you must restrict the extension of the volume in z-direction in a way that the undoped end caps are not contained in the volume.

For the entries the position of the origin as defined in help item "Models" has to be taken into account correctly.

In case of models 2 and 4 (pump light distributions with top hat profile along z-axis) the entries are used to truncate the distribution in x, y, or z-direction.

If the option button "**Two materials ...**" is checked an additional frame is shown with caption "**Material A extends**" that displays two option buttons. Material A can extend

in a rectangular (or brick shaped) region. The limits of this region can be defined in the same way as the limits of the pumped region but must not be identical with the latter ones. Alternatively, material A can extend in a cylindrical region parallel to the z-axis. To define this region the radius of the cylinder, x and y coordinate of the cylinder axis, as well as left and right end of the cylinder must be entered.

6.10.6 Tab "FEA Options"

The entries for this tab control the mesh and the computational algorithm of the the FEA code. In addition, the directory can be defined where the result files generated by the FEA code are stored.

As described in section 4.2, the FEA code uses a fine equidistant mesh inside the crystal. The element edge length perpendicular to the crystal axis is defined by "**Mesh size in x- and y-direction**"; the element edge length parallel to crystal axis is defined by "**Mesh size in z-direction**".

To obtain an estimate for the number of elements being generated the button "**Estimated number of elements**" can be pressed. See section 1 for memory considerations.

Usually the entries for the element size are changed slightly after pressing the "**Apply**" button due to adjustments carried through to fit the element size to dimensions of crystal or doped regions. Since in many cases the relevant physical quantities change much slower in longitudinal than in transverse direction it is advisable to define a coarser mesh in z-direction to reduce the computational effort. Additional information about mesh size, element numbers and FEA results can be found in the file `output.txt` written into the selected FEA directory.

The "**Convergence limits**" control the convergence of the iterative computation. The default value $1.0E-7$ for the thermal analysis stops the code if the maximum temperature does not change within the first 7 digits. The limit for the structural analysis refers to the absolute value of maximum nodal displacement.

The entries for the "**Maximum number of iterations**" stop the iteration process regardless if the convergence limit is met or not.

The graphics control input "**Position of cutting plane perpendicular to z-axis**" allows the placement of a cutting plane perpendicular to the z-axis to show the distribution of physical quantities inside the crystal at this plane with the **3D Visualizer**. If this input is set to zero, which is the default setting, the cutting plane is identical with the x-z-plane. If the input is greater than the length of the crystal the full crystal is shown.

Press the button "**Apply and run FEA**" to transfer the entries to internal variables, and to start the FEA code. A window is popping up showing the progress of computation. In addition to the current number of iteration, it shows the maximum temperature for thermal analysis, and the absolute value of maximum nodal displacement for structural analysis. The button "**Skip**" allows the user to stop thermal or structural analysis at the actual iteration step and to proceed to structural analysis or to finish computation, respectively. At the end of the computation the message "FEA finished successfully"

appears, press the "OK" button to close the dialog. Please be aware that initialization of the FEA and generation of the mesh can take some time for large element numbers.

The results of FEA are written into files in the directory defined in the box with caption "**Directory for output of FEA results**".

For 3D graphical visualization of the FEA results click "**FEA/3D Visualizer**" in the main LASCAD menu to open the related window, which is described in the next section. For 2D visualization in form of diagrams click the menu item "**2D Data Profiles and Parabolic Fit**".

6.11 Window "3D Visualizer"

Menu and buttons of this window are almost self-explaining.

Use the dropdown button leftmost in the toolbar of this window to select a physical quantity like heat load, temperature etc. to be displayed.

The data used for the plots are taken from the FEA results default directory that is shown in the row below the toolbar. Use the button at the end of this row to switch to an other directory. To open graphic files from arbitrary directories use the menu item "**File/Open Datafile...**".

Use the left mouse button to rotate an object. Longitudinal motion of the mouse causes rotation about the object x-axis, transverse motion causes rotation about the object y-axis. Use the middle mouse button to move the object in the plane of the screen, and the right button to zoom the object (longitudinal mouse motion), or to rotate it around the object z-axis (transverse motion). If you have a 2-button mouse, press the button showing crossed double arrows, and then use the left button to move the object in the plane of the screen.

The button showing the floppy disk symbol allows saving the actual plot as Windows Bitmap and corresponds to the menu item "**File/Save as Bitmap**". The next 6 buttons allow to show front, back, side, top, and bottom view of the object and correspond to the menu items "View/Front", "View/Back" etc.

The two buttons with red "X" and "Y" and vertical arrow are related to the left mouse button and allow rotation about object x-axis or y-axis only with y-axis or x-axis being locked, respectively.

The two buttons showing red "Z" or two squares are related to the right mouse button and allow to rotate about the object z-axis or to zoom only, respectively. The next button showing crossed double arrows has already been explained.

The button with the "wire frame slab" icon is useful for big data sets; it allows to rotate a wire frame instead of the full object, and to show the object only after releasing the mouse button. If this button is pressed a mouse click into the plot shows the origin of the coordinate system with the XYZ triad.

The next button to the right allows to toggle the background from black to white that is useful for printing a plot. The button with the bulb allows to switch on or to turn off illumination.

All 6 Cartesian components of the stress tensor can be shown. They are designed by "stress xx (yy, zz, xy, xz, yz) comp.". Additionally, the von Mises equivalent stress σ_{eqv} defined by

$$\sigma_{eqv} = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}, \quad (11)$$

and the so called stress intensity σ_I defined by

$$\sigma_I = \max(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) \quad (12)$$

can be shown. Here σ_1 , σ_2 , and σ_3 are the principal stress components obtained by diagonalization of the stress tensor. As explained in textbooks of structural mechanics especially the stress intensity represents an important measure for stress load and crack formation in the crystal, though σ_{eqv} and σ_I usually do not differ much.

The menu item "**View/Reset Graphics**" resets all settings to default values. The menu item "**View/Rotation Angles**" opens a window showing the actual rotation angles and the zoom factor. Into this window values can be entered numerically to rotate or zoom the plot.

The slider "**mouse speed**" can be used to adjust the relation between speed of rotation and mouse motion

6.12 Window "Pump Profile"

To open this window click the tab "**Pump Light**" of the window "**Crystal, Pump Beam, and Material Parameters**" and press the button "**Show Pump Profile**", as described in section 6.10.2.1. Alternatively you can click "**View/Pump Profile**" in the menu of the main LASCAD window.

The two graphs show the profile of the absorbed pump power density in W/mm³ along two axes parallel to x- and y-axis which intersect at the peak of the transverse profile. To get correct profiles the absorption coefficient must be defined in advance in the tab "Material Parameters" of the window "Crystal, Pump Beam, and Material Parameters".

Be aware that **different from the present window the 3D Visualizer for the FEA results** is showing the **fractional thermal load** which is obtained by multiplying the absorbed pump power density with the "**Heat efficiency factor**" as defined in the tab "Material Parameters" of the window "Crystal, Pump Beam, and Material Parameters".

The position of the cutting plane along the crystal axis where the profiles are shown can be changed by the use of the slider. The distance of this position from the left face of the crystal is given in the box below the graphs, where this parameter also can be entered directly.

The vertical scale of the graphs can be changed by the use of the spin button or by entering a value into the related box. A mouse click into one of the graphs fits the height of the graphs to the height of the diagram.

6.13 Window "2D Data Profiles and Parabolic Fit"

To open this window click the menu item "FEA/2D Data Profiles and Parabolic Fit" in the main LASCAD window. Before the window is opened a dialog is popping up with a directory tree in order to select the directory with the FEA results, since the name of this directory can be defined by the user optionally.

The Window "2D Data Profiles and Parabolic Fit" can be used to display 2D profiles of FEA results along x- and y-axis at cross sections which cut the z-axis at the grid points created by the FEA discretization.

Use the drop down box "Select z-axis value ..." in the upper right corner of the window to select a grid point along the z-axis. Highlight the drop down box, and use the mouse wheel to scroll to different positions along the z-axis, and to show the related diagrams simultaneously.

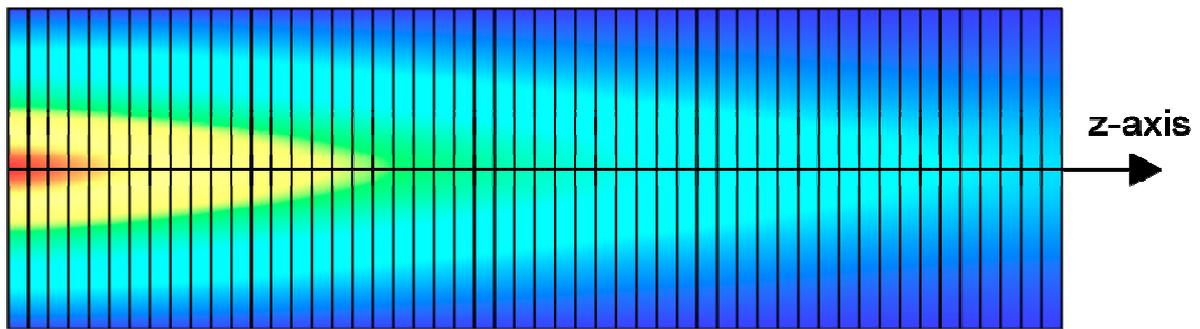
Use the drop down box "Show Item" in the frame "FEA Data" to select an item to be shown. You can select from temperature, absorbed pump power density, z displacement, diagonal components of the stress tensor, equivalent stress, and stress intensity.

To generate the diagrams the file `FEA_x_y.out` created by the FEA code is used which therefore must exist in the FEA directory. The file contains the FEA results together with the parameters shown in the window "Crystal, Pump Beam,..." when the FEA has been started. If you open the window "2D Data Profiles and Parabolic Fit" or press the button "Refresh Data" in this window, all these parameters including material parameters are reset to the values which has been used with the related FEA run. Excepted from this rule are the parameters refractive index and derivative of refractive index versus temperature which are not reset, to give the user the possibility to modify these parameters without running the whole FEA.

To carry through a parabolic fit of the profiles press the button "Refresh & Fit". For the fit only nodal points are used which are located around the crystal axis within a range defined by the entries in the drop down boxes "x-Range of Fit" and "y-Range of Fit". You can select a value from the list or enter a new one. The ranges should be once to twice as large as the expected mode spot size. After completing the fit the plots show the result along x- and y-axis. To visualize deviations between fit and FEA data the diagrams show a range that is twice as large as the range used for the fit. Again you can show the fit at different positions along the z-axis by the use of the drop down box "Select z-axis value ...". The fit diagram can be shown for the items refractive index, left and right face of crystal and pump profile which can be selected from the drop down box "Show Parabolic Fit" in the frame "Parabolic Fit". For each diagram the parabolic coefficients belonging to the x-z- or the y-z-plane, respectively, are shown in the 2 boxes in the frame "Parabolic Coefficients".

The parabolic coefficients correspond to the second derivative of the related profile averaged within the range of fit and multiplied by (-1), and are in agreement with corresponding parameters defined in *Siegman, LASERS*, Chap. 20. Their use in the ABCD matrix algorithm can be described in the following way.

By the FEA discretization the laser crystal is subdivided into thin layers perpendicular the z-axis as shown in the picture.



For each of these layers an ABCD matrix is computed by the use of the parabolic coefficients obtained by the fit. All these matrices together with the matrices for the end faces of the crystal and additional elements in the cavity are multiplied with each other to compute the full round trip ABCD matrix. The latter finally allows for computing the mode shape (see e.g. *A.E. Siegman, LASERS, Chap. 21.1*). For this computation the actual values for **the refractive index and its derivative versus temperature** shown in the tab "Material Parameters" of the window "*Crystal, Pump Beam, and Material Parameters*" are used. **Therefore, it is important to enter the desired values here before carrying through the fit.**

After carrying through the parabolic fit the **focal length of the thermal lens** is shown at the bottom of the window. It is measured as distance from the left or right principal plane of the thermal lens, respectively.

Simultaneously with the plots of the fit a small yellow window is popping up. According to the instructions given in this window, press the ALT key and click in the mode plot into the field where the crystal shall be inserted. A yellow-ocher colored symbol for the thermally lensing crystal is being inserted. The length of the field is adjusted to the length of the crystal as defined in the tab "**Models**" of the window "**Crystal, Pump Beam, ...**". The elements at both ends of the field are transformed into the end faces of the crystal. If you prefer that existing elements are not being transformed you must insert dummy elements in advance.

Simultaneously with the insertion of the crystal into the mode plot the file FIT.dat containing the parabolic coefficients is written to the selected FEA directory. In the same directory two other files GETFEAData.txt and FitFEAData.txt generated by the present dialog can be found that contain numerical data of the nodes along the z-axis and of the transverse profiles.

To clear the thermal lens in the mode plot window, press the CTRL key and click with the left mouse button into the thermal lens (see manual section 6.2.1).

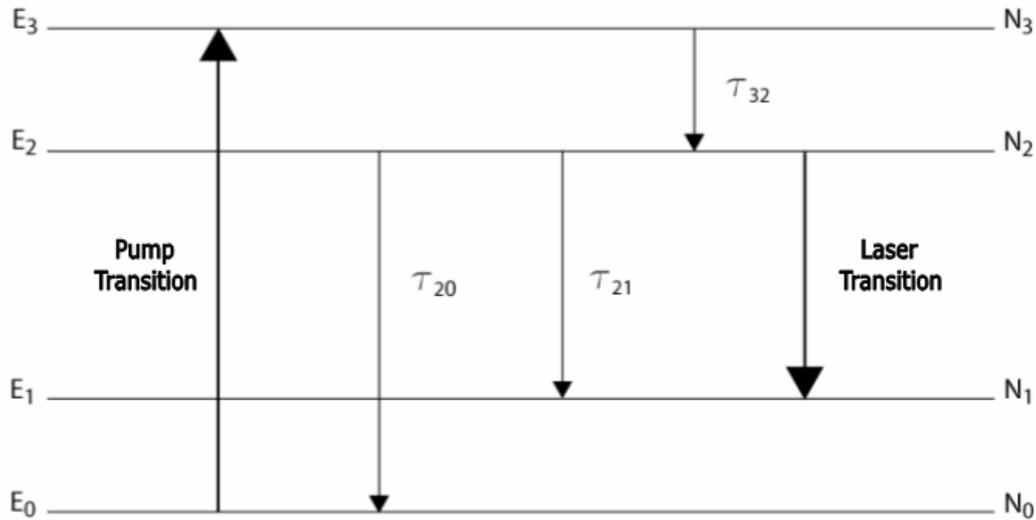
The button "**Refresh Data**" reads the actual FEA results from the FEA directory and should be used after a new FEA run.

The button "**Print**" can be used to produce a plot of the diagrams.

6.14 Window "Laser Power Output"

To compute the laser power output the rate equations have to be solved. Laser power output for 4-level systems as well as for 3-level systems can be computed.

In a **4-level-laser system** it is assumed that the distance between lower laser level and the ground level is large enough that the thermal occupation of the lower laser level can be neglected that means $N_1 \sim 0$. In addition, a fast transition between E_3



Energy levels, population densities and transitions in a 4-level-laser system

and E_2 is assumed. Therefore, for the description of a 4-level-system the following simplified the rate equations can be used

$$\frac{\partial N}{\partial t} = R_p - WN - \frac{N}{\tau} \quad (13)$$

$$\frac{dS_L}{dt} = \iiint_a WN dV - \frac{S_L}{\tau_c} \quad (14)$$

Where

$N(x,y,z) = N_2 - N_1$	population inversion density ($N_1 \sim 0$),
$R_p = \eta_p P_a / h\nu_p$	pump rate,
η_p	pump efficiency,
$P_a(x,y,z)$	absorbed pump power density,
$h\nu_p$	pump photon energy,
$W(x,y,z)$	transition rate due to stimulated emission,
τ	spontaneous fluorescence life time of upper laser level,
S_L	number of laser photons in the cavity,
τ_c	mean life time of laser photons in the cavity.

The 3-D integration extends over the crystal volume.

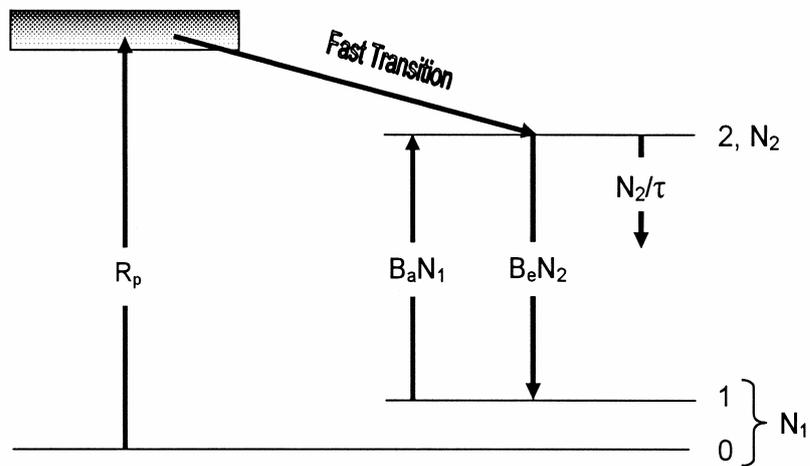
For a **Quasi-3-level system**, as shown in the figure below, the rate equations are given by

$$N_t = N_1 + N_2 \quad (15)$$

$$\frac{\partial N_2}{\partial t} = R_p - (B_e N_2 - B_a N_1) - \frac{N_2}{\tau} \quad (16)$$

$$\frac{\partial S_L}{\partial t} = \iiint_a (B_e N_2 - B_a N_1) dV - \frac{S_L}{\tau_c} \quad (17)$$

The 3-D integration again extends over the crystal volume.



Quasi-3-level scheme

The individual terms in Eqs. (15) to (17) are defined as follows:

N_t doping density per unit volume

B_e transition rate for stimulated emission

$$B_e = \frac{c\sigma_e}{n} S_L s_0(x, y, z) \quad (18)$$

B_a transition rate for reabsorption

$$B_a = \frac{c\sigma_a}{n} S_L s_0(x, y, z) \quad (19)$$

where

σ_e	effective cross section of stimulated emission
σ_a	effective cross section of reabsorption
c	the vacuum speed of light
n	refractive index of laser material

$s_0(x,y,z)$ is the normalized distribution of laser photons in the cavity i. e.

$$\iiint_R s_0(x, y, z) dV = 1 \quad (22)$$

The cross section of stimulated emission is being computed by the use of the method of reciprocity. As shown in the paper of Laura L. DeLoach et al. , IEEE J. of Q. El. **29**, 1179 (1993) the following relation can be deduced

$$\sigma_e = \sigma_a \frac{Z_l}{Z_u} \exp\left(\frac{E_{zL} - h\nu}{kT(x, y, z)}\right) \quad (23)$$

Here Z_u and Z_l are the partition functions of the upper and lower crystal field states, E_{zL} is the energy separation between lowest components of the upper and the lower crystal field states. k is Boltzmann's constant, and $T(x,y,z)$ [K] is the temperature distribution in the crystal as obtained from FEA. Also Z_u and Z_l depend on the local temperature.

The parameters needed for the computation of power output of 3-level-systems can be defined in the window "**Parameters for Quasi-3-Level Laser**" as described in section 6.15.

To solve the rate equations for the steady state case an iterative integration procedure is being applied, which uses the FEA mesh generated for thermal analysis. A detailed description of the numerical procedure will be given in a separate tutorial.

Description of the inputs and buttons of the window "Laser Power Output":

The frame "**Plot (Options)**" contains three option-buttons which allow selecting from the following 3 modes of plotting:

Curve with ... grid points: A series of results is being computed over equidistant grid points within an interval along the x-axis whose limits are defined by "**X-scale min.**" and "**X-scale max.**" To modify the number of grid points change the number in the corresponding box. If the x-axis variable corresponds to the pump power the results of thermal FEA are linearly rescaled proportional to the pump power (see below).

Single Point: The laser power output is being computed for a single set of input parameters, which can be defined by the use of the input boxes above the diagram.

Experimental Data: This option allows reading and plotting of data from a file that contains two columns of experimental data separated by white-space for pump power (or reflectivity of output mirror) and laser power (right column), respectively.

The frame "**Define X-Axis Variable**" contains three option buttons to define the X-variable:

The options "**Use incident pump power**" and "**Use absorbed pump power**" are selected by default. "**Use incident Pump power**" is used with cavity configuration models 1, 3, 5, and 6 as provided in the window "Crystal, Pump Beam, and Material Parameters", and "**Use absorbed pump power**" with cavity configuration models 2, 4, 7, and 8, respectively. Alternatively, in both cases the output mirror reflectivity can be used as x-variable. The latter also holds for plots of experimental data. Use of other resonator parameters as x-axis variable is under development.

For pump power values, which are different from the pump power initially used for the thermal analysis, the results of the FEA are rescaled linearly. This delivers correct results as long as the material parameters used for the FEA are independent of temperature. In case of temperature dependent material parameters, the FEA has to be repeated for every pump power value. The latter also holds if boundaries of the crystal have different temperatures.

The button "**Apply**" confirms the data input for use with computation.

Pressing the button "**Apply & Plot**" computes the laser power output and plots the computed data dependent on the selected options. It may happen that plotted items are outside the range of the diagram. In this case, press the button "**Adjust Scale**" to adjust x- and y-scale limits to the extremal values of the data to be plotted.

The button "Redraw" plots the data set within the limits defined in the boxes "X-scale min.", "X-scale max.", "Y-scale min.", and "Y-scale max."

Use the "**HotHit**"-functionality of the plot to show numerical data for individual grid points shown in the plot. If you click into a point marker (small circle), its color changes

to red, and the related numerical values are shown in the three boxes in the lower part of the right hand side panel of the window.

"**X-Axis Variable**" shows the corresponding abscissa value.

"**Laser power**" shows the laser power output.

"**Absorbed Pump Power**" shows the pump power totally absorbed in the crystal.

The 4 boxes above the plot are almost self-explaining:

"**Resonator round-trip loss**" represents the combined losses that in addition to output coupling the wave experiences during a round-trip in the cavity. This means losses due to diffraction, scattering, absorption, limited reflectivity of highly reflecting mirrors etc.

"**Incident (or Absorbed) pump power**" must be used to define incident or totally absorbed pump power for a single point plot.

Check the box "**Account for Apertures**" to take into account aperture settings which can be defined by opening the tab "Apertures" in the window "Parameter Field". Independent of aperture settings and the status of said box the lateral surfaces of the crystal always are taken into account for the computation of laser power output. The status of said box does not change the profile of the laser mode.

Check the box "**Multimode Operation**" to approximate a multimode laser beam by the use of a top-hat profile which is defined in a way that the beam has uniform intensity different from zero within an elliptical region transverse to the propagation direction. The z-dependent main axes of this elliptical region are defined by the multimode spot sizes $w_{mx} = \sqrt{M_x^2} w_{fx}$ and $w_{my} = \sqrt{M_y^2} w_{fy}$, where $w_{f x(y)}(z)$ and $M_{x(y)}^2$ are fundamental spot sizes and beam quality parameters for x-plane and y-plane mode, respectively. If the box "Account for Apertures" is checked beam quality must not be defined explicitly, w_{mx} and w_{my} are computed automatically corresponding to aperture settings. If the box "Account for Apertures" is unchecked, the tab "Miscellaneous" in the window "Parameter Field" must be opened to define beam quality.

Check the box "**Pulsed Operation**" to account for of pulsed operation. This is done approximately by compressing the continuously applied pump power to a series of short rectangular pulses corresponding to pulse length and frequency. A more realistic analysis based on the related differential equation is under development. The obtained result for the laser power output is the time averaged value.

The thermal FEA-file used with the actual computation of laser power is shown in the caption of the window.

6.15 Window "Parameters for Quasi-3-Level Lasers"

This window can be opened from the main menu by clicking "View/3_Level_Material Parameters" or by opening the tab "Material Parameters" of the window "Crystal,

Pump Beam, and Material Parameters" and clicking the button "Show Material Parameters for 3-Level-Systems".

The entries allow for defining the following material parameters needed for the computation of laser power output of 3-level-systems:

- doping density,
- effective cross section of reabsorption,
- energy levels of crystal field states,
- degeneration of crystal field states.

The window also contains two option buttons to select between temperature dependent computation of the stimulated emission cross section according to Eq. (23), or direct use of the corresponding input in the index card "Material Parameters" of the window "Crystal, Pump Beam, and Material Parameters". In the second case temperature dependence of the stimulated emission cross section is neglected.

6.16 Window "Dynamic Multimode Analysis (DMA) and Q-Switch Modeling"

Generate Input and Open GUI for DMA Code saves the cavity configuration to a file which is used as input for the DMA code, and opens the GUI of the DMA code.

Generate Modified Input for DMA only saves the cavity configuration to a file, but does not open the DMA GUI. The button can be used to update the cavity configuration, if the DMA GUI is already open.

Open DMA Viewer can be used to visualize results of previous DMA runs. After opening the DMA Viewer press **Change...** to select a DMA directory.

Close closes the DMA activation window, but not the GUI and other windows of the DMA code.

7 Dynamic Multimode Analysis (DMA)

7.1 Introduction

The purpose of the Dynamic Multimode Analysis is to analyze multimode and Q-switch operation of lasers. To this end the transverse mode structure in the cavity is approximated by a set of M Hermite-Gaussian (HG) or Laguerre-Gaussian (LG) modes. Since HG and LG modes are representing sets of orthogonal eigenfunctions with different eigenfrequencies, it is assumed that each transverse mode oscillates independently, and therefore the influence of short-time interferences between the modes can be neglected on average. Based on this assumption, the interaction between the population inversion density and the photon numbers in the oscillating modes can be described by a set of time dependent 3D rate equations. These equations deliver time dependent results for the photon numbers in the individual modes, which are used to describe their contribution to the whole transverse mode structure. In this way, a time dependent analysis of mode competition effects and multimode behavior of the cavity is obtained. This result is used to compute beam quality and laser power output, and to describe time dependence of Q-switch operation.

7.2 Multimode Rate Equations

Based on the assumptions made in the introduction, the multimode behavior of the cavity is described by the following set of time dependent 3D rate equations

$$S_C = \sum_{i=1}^M S_i \quad i=1, \dots, M, \quad (7.1)$$

$$\frac{\partial S_i}{\partial t} = \frac{c\sigma}{n_A} \int_{\Omega_A} N S_i s_i dV - \frac{S_i}{\tau_C}, \quad (7.2)$$

$$\frac{\partial N}{\partial t} = -\frac{c\sigma}{n_A} N S_C S_C - \frac{N}{\tau_f} + R_p \frac{N_{dop} - N}{N_{dop}} \quad (7.3)$$

which describe the interaction between the population inversion density $N(x,y,z,t)$, the total photon number $S_C(t)$ in the cavity with associated normalized photon density distribution $s_C(x,y,z,t)$, and the photon numbers $S_i(t)$ belonging to the individual transverse modes with associated normalized photon density distributions $s_i(x,y,z,t)$ of the individual modes.

In Eqs.(7.2 and 7.3) the following parameters are used:

n_A	refractive index of the active medium,
c	vacuum speed of light,
$N(x,y,z,t) = N_2 - N_1$	population inversion density ($N_1 \sim 0$),
$R_p = \eta_p P_a / h\nu_p$	pump rate,
η_p	pump efficiency,
$P_a(x,y,z)$	absorbed pump power density,
σ	effective cross section of stimulated emission,
τ_C	mean life time of laser photons in the cavity,
τ_f	spontaneous fluorescence life time of upper laser level,
N_{dop}	doping density.

Eqs. (7.2 and 7.3) refer to a 4-level-system. Multimode analysis of quasi 3-level systems is under development. Fast decay rates between level 3 (pump level) and 2 (upper laser level), and between level 1 (lower laser level) and 0 are assumed.

To obtain the normalized photon densities s_i ($i=C; 1, \dots, M$) the complex wave amplitudes $u_i(x, y, z)$ are normalized over the domain $\Omega = \Omega_{2D} \times [0, L_R]$ of the resonator with length L_R . Here the u_i ($i=1, \dots, M$) denote the amplitudes of the individual modes, whereas u_C denotes the amplitude of the superposition of these modes whose absolute square in our incoherent approximation is given by

$$|u_C(x, y, z)|^2 = \sum_{i=1}^M |u_i(x, y, z)|^2 . \quad (7.4)$$

To carry through the normalization we define mode volumes V_i by the relation

$$V_i = \int_{\Omega_A} n_A |u_i|^2 dV + \int_{\Omega/\Omega_A} |u_i|^2 dV , \quad (7.5)$$

where Ω_A is the domain of the active medium. By the use of the V_i the $|u_i|^2$ can be normalized in the following way

$$s_i = \begin{cases} \frac{n_A}{V_i} |u_i|^2 & \text{inside the crystal} \\ \frac{1}{V_i} |u_i|^2 & \text{outside the crystal} \end{cases} \quad (7.6)$$

This normalizes the s_i to

$$\int_{\Omega} s_i = 1 .$$

The discontinuous definition of s_i is necessary to take into account the different photon densities inside and outside the active medium due to the different speed of light. If, in addition to the active medium, other media with refractive indices >1 are present in the cavity, Eqs. (7.5 and 7.6) are modified accordingly.

In case of standing wave resonators superposition of the waves u_R and u_L propagating to the right and to left must be taken into account by

$$|u_i|^2 = |u_{iR}|^2 + |u_{iL}|^2 . \quad (7.7)$$

To compute τ_c the overall resonator losses are defined by

$$L_{Res} = L_{rtrip} - \ln(R_{out}) , \quad (7.8)$$

which is the sum of internal round trip losses L_{rtrip} , for instance scattering, absorption and diffraction, and the loss $-\ln(R_{out})$ at the output coupler, where R_{out} is the reflectivity of the output mirror. The time for the roundtrip is given by

$$t_{rtrip} = \frac{2\tilde{L}}{c} \quad (7.9)$$

Here \tilde{L} the optical path length of the cavity which is given by

$$\tilde{L} = L_R + (n - 1)L_A, \quad (7.10)$$

if the active medium is the only medium with refractive index > 1 . From Eqs. (7.8 and 7.9) we obtain for τ_c the expression

$$\tau_c = \frac{t_{rtrip}}{L_{Res}} = \frac{2\tilde{L}}{c(L_{roundtrip} - \ln(R_{out}))}. \quad (7.11)$$

Equations (7.2) and (7.3) are discretized in space and time. Since neither equation contains spatial derivatives, it is enough to apply a simple finite volume discretization in space. In this way, a stable time discretization is obtained by a stiff solver for ordinary differential equations.

7.3 Apertures and Mirrors with Variable Reflectivity

Apertures and output mirrors with variable reflectivity can be taken into account in Dynamic Multimode Analysis by introducing specific losses L_i for the individual modes. This leads to mode specific mean life times $\tau_{c,i}$ of the photons, and in consequence to the following modified rate equations for the photon numbers

$$\frac{\partial S_i}{\partial t} = \frac{c\sigma}{n} \int_{\Omega_A} NS_i s_i dV - \frac{S_i}{\tau_{c,i}}. \quad (7.12)$$

In case of an aperture with radius R_A at position z_A and a mirror with uniform reflectivity, the mode specific losses are described by

$$L_i = L_{rtrip} - \ln \left[R_{out} \int_0^{R_A} \int_0^{2\pi} \tilde{s}_i(r, z_A) 2\pi dr d\varphi \right]. \quad (7.13)$$

Here $\tilde{s}_i(r, z_A)$ is the photon distribution of mode i at position z_A normalized with respect to the transverse coordinates, i. e.

$$\tilde{s}_i(x, y, z_A) = s_i(x, y, z_A) / \iint s_i(x, y, z_A) dx dy \quad (7.14)$$

If the position of the output coupler is not identical with z_A , the aperture R_A is rescaled according to the ratio of gaussian spot sizes at the position of the output coupler and the aperture. The losses caused by rectangular apertures are described analogously to Eq.(7.13).

In case of a mirror with variable reflectivity, for instance a gaussian mirror, the mode specific losses are described by

$$L_i = L_{rtrip} - \ln \left[\iint R_{out}(x, y) \tilde{s}_i(x, y, z_A) dx dy \right]. \quad (7.15)$$

The mode specific mean life times $\tau_{c,i}$ of the photons are related to the mode specific losses L_i according to

$$\tau_{c,i} = \frac{t_{rtrip}}{L_i} \quad (7.16)$$

Eqs. (7.12 to 7.16) describe the effect of apertures and mirrors with variable reflectivity only approximately. In reality, apertures and mirror reflectivity affect the shape of the modes. This can not be taken into account in the gaussian mode algorithm. However, the approximation is not really bad, since the shape of higher order transverse modes is not important, if they are suppressed by apertures. Otherwise, the effect on the fundamental mode is only important, if the aperture is smaller than the mode diameter, which in real lasers usually is not the case. Variable reflectivity is taken into account in the gaussian algorithm in case of gaussian mirrors.

7.4 Laser Power Output

The laser power output is obtained by computing the number of photons passing the output coupler per time unit. In this way one obtains for the power output delivered by the individual transverse modes

$$P_{i,out}(t) = h\nu_L S_i(t) \frac{-\ln(R_{out})}{t_{rtrip}} [1 - 0.5\ln(R_{out})], \quad (7.17)$$

The factor $1-0.5*\ln(R_{out})$ takes into account that the photon density in front of the output coupler is approximately a factor of this size higher than the mean photon density. The total power output is given by adding up the power output of the individual modes

$$P_{C,out}(t) = \sum_{i=1}^M P_{i,out}(t) \quad (7.18)$$

The power output averaged over a time period t_1 until t_2 is computed as

$$P_{i,average} = \frac{1}{(t_2 - t_1)} \int_{t_1}^{t_2} P_{i,out}(t) dt, \quad (7.19)$$

If an aperture is present Eq. (7.20), must modified into

$$P_{i,out}(t) = h\nu_L S_i(t) \frac{R_{log,i}}{t_{rtrip}} [1 - 0.5R_{log,i}], \quad (7.20)$$

Here the logarithmic loss $R_{\log,i}$ is given by

$$R_{\log,i} = -\ln \left[R_{out} \int_0^{R_A} \int_0^{2\pi} \tilde{s}_i(r, z_A) 2\pi r dr d\phi \right] \quad (7.21)$$

in case of an aperture, and by

$$R_{\log,i} = -\ln \left[\iint R_{out}(x, y) \tilde{s}_i(x, y, z_A) dx dy \right] \quad (7.22)$$

in case of an output mirror with variable reflectivity, respectively.

7.5 Beam Quality

The beam quality factors M_x^2 and M_y^2 are computed according to Siegman and Townsend*) using the expressions

$$M_x^2(t) = \sum_{i=1}^M (2p_i + 1) c_i(t) \quad (7.23)$$

and

$$M_y^2(t) = \sum_{i=1}^M (2q_i + 1) c_i(t) . \quad (7.24)$$

Here p_i and q_i are the transverse mode orders of the i -th gaussian mode in x - and y -direction, respectively. The coefficients $c_i(t)$ are defined by

$$c_i(t) = \frac{P_{i,out}(t)}{P_{C,out}(t)} , \quad (7.25)$$

where $P_{i,out}(t)$ and $P_{C,out}(t)$ are given by Eqs.(7.17, 7.18, and 7.20), respectively.

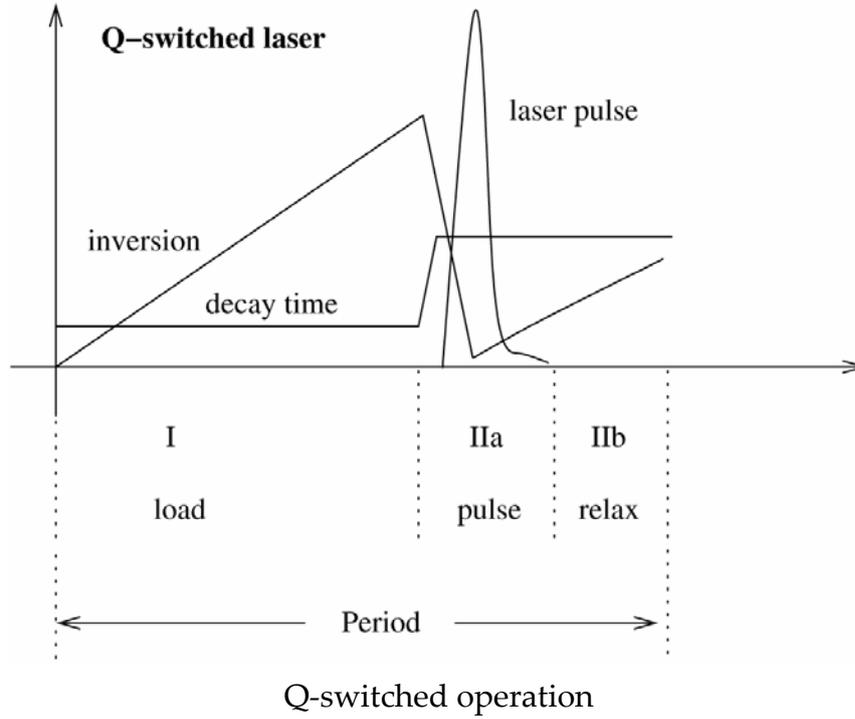
*) A.E. Siegman and S. Townsend, "Output beam propagation and beam quality from a multimode stable-cavity laser", IEEE Journal Quantum Electron., **29**, 1212–1217 (1993)

7.6 Q-Switching Analysis

Time dependence of Q-switching is characterized by three time periods which can be described as follows:

- load period – period I
- pulse period – period IIa
- relaxation period – period IIb

Development of population inversion and laser power during these periods is shown schematically in the figure "Q-switched operation".



In the first period the active material is pumped to a high level of population inversion. During this period the lasing process is prevented by introducing a high artificial intra-cavity loss, which can be generated by different methods.

To describe the first period mathematically, it is assumed that the photon number in the individual modes vanishes. This simplifies the rate equations to

$$\frac{\partial S_i}{\partial t} = 0, \quad i=1, \dots, M \quad (7.26)$$

$$\frac{\partial N}{\partial t} = -\frac{N}{\tau_f} + R_p \frac{N_{dop} - N}{N_{dop}} \quad (7.27)$$

Consequently, during the first period, the population inversion is limited by the doping density of the lasing atoms N_{dop} and the spontaneous fluorescence life-time of the upper laser level.

After the load period the artificial intra-cavity loss is removed, and the generated high population inversion is depleted by a short laser pulse of high power. This is time period IIa. Time period IIb is a waiting period until the whole Q-switching process is repeated. Time periods IIa and IIb are modeled by the original rate equations (7.2) and (7.3). In case of active Q-switching the overall process is actively triggered with high constant frequency, in case of passive Q-switching it is assumed that single pump pulses are applied whose intensity is constant during the load period.

To get an improved approximation, it can be assumed, that the opening time t_{open} of the Q-switch is finite. This means that a finite time period t_{open} elapses until the Q-switch

becomes transparent after being triggered, for instance $t_{\text{open}} = 50$ ns. During t_{open} the overall cavity loss is reduced to the normal cavity loss L_{Res} .

7.6.1 Pulse Shape

A useful parameter to characterize the pulse shape is proposed in the textbook A.E.Siegmann, LASERS, University Science Books, Chapt. 26 as

$$r_{\text{Pulse}} = \frac{N_L}{N_{\text{th}}} \quad (7.28)$$

To evaluate the pulse shape parameter r_{Pulse} in our 3D model for N_L , the maximum value of population inversion N at the crystal axis at the end of the load period is used. For N_{th} the threshold value for the cavity without Q-switch loss is used, assuming a uniform distribution of N . According to Siegman, a smaller value of r_{Sieg} corresponds to a broader pulse shape, whereas with increasing r_{Sieg} , the pluses become sharper and asymmetrical.

7.6.2 Numerical Discretization

The time periods I, IIa, and IIb are completely different from a numerical point of view. The rate equations in period I are simple differential equations, due to the assumption $S_i=0$. In period IIa, however, the function of the photon density has very large derivatives. Therefore, large time steps can be used in period I and IIb, whereas very small time steps are necessary in period IIa. Consequently, the solution for period I is obtained by a few, fast iterations. Considering the fact, that period I is much larger than IIa and IIb, this leads to a very fast computation of the Q-switch pulse.

7.7 The GUI of the DMA Code

Parameters to be defined in the graphical user interface of the DMA code

7.7.1 Tab "Gaussian Modes"

Type of gaussian modes: *Hermite-* or *Laguerre-*Gaussian modes. The option *Automatic* selects HG or LG modes depending on the astigmatism of the laser mode structure. See astigmatism threshold.

Astigmatism threshold for automatic mode type selection: The astigmatism threshold is defined as the maximum relative deviation of the spot sizes w_x and w_y of the laser mode along the z-axis in percent:

$$\max_z \left| \frac{2(w_x(z) - w_y(z))}{w_x(z) + w_y(z)} \right|$$

Automatic computation of maximum mode order: If enabled, the maximum mode order is automatically determined, dependent on the averaged pump beam radius.

Maximum transverse mode order: Defines the highest transverse mode order taken into account in x- as well as in y-direction. If the maximum transverse mode order is N_{\max} , the total number M of modes taken into account is

$$M = \sum_{n=0}^{N_{\max}} (2n + 1) = N_{\max}^2 + 2N_{\max} + 1 . \quad (7.29)$$

The maximum mode order is limited to 12.

Open the menu item **Show additional items/Transverse Gaussian Mode Profile** in the mode plot window of LASCAD to show the overlap between pumplight distribution and higher transverse modes.

Number of grid points in x- and y-direction: Defines the number of grid points used to discretize the transverse computational domain in x- as well as in y-direction. In order to represent higher order transverse modes, especially LG modes with sufficient resolution, the number of transverse grid points should be increased as the maximum mode order increases.

Number of grid points in z-direction: Defines the number of grid points used to discretize the computational domain along crystal axis.

Stretch factor in x- and y-direction related to beam diameter: By default the transverse width of the computational domain is adjusted to the maximum $1/e^2$ diameter of the profile of the highest order transverse mode taken into account. "Maximum" means the maximum mode diameter along the crystal axis. It is recommended to define a stretch factor >1 to increase this width. For instance, in case of the TEM₀₀ mode, a factor >2.5 or even higher dependent on the special pump light distribution is recommended. For higher order modes a smaller stretch factor can be used. The transverse width of the computational domain finally used for computation is written to the file `output.txt` which can be found in the DMA working directory. Dependent on the cavity configuration the transverse width of the computational domain can have a considerable effect on the contribution of the individual modes to the laser power output, since it has an effect similar to an aperture.

7.7.2 Tab "Rate Equations"

Resonator round-trip loss: Overall internal round trip losses, for instance: scattering, absorption and diffraction, corresponds to L_{rtrip} in Eq. (7.8).

Reflectivity of output mirror: Currently, the output coupler is the right end mirror of the cavity. Reflectivity is the relative amount of radiation reflected back into the cavity.

Wavelength of pump light: self-explanatory

Doping density [Ions/mm³]: Has to be defined consistent with the absorption coefficient for the pump light defined in the tab **Material Parameters** of the FEA window (see section 6.10.4). By default the value 1.37E17 corresponding to 1 atomic % Nd:YAG is used.

To carry through a comparison with the results obtained with the tool for time independent computation of **CW Laser Power** the parameters in the window **Laser Power Output** must be defined consistent with the above ones. There is no internal transfer of these parameters between the computational tools.

Other parameters used in the rate equations must be defined in the tab **Material Parameters** of the FEA window. The pump rate is computed by the use of the 3D absorbed pump power density written to the file FEA.out by the FEA code.

7.7.3 Tab "CW Operation"

Time resolution: Defines the discretization of the time period used to solve the rate equations.

Time period used for simulation [s]: self-explanatory

Number of time steps: Number of time steps obtained by dividing the time period by the time resolution. This value has a critical effect on the computational time.

7.7.4 Tab "Q-switch"

Q-switch analysis: On/Off

The current version of the code supports cw and pulsed pumping. If enabled, parameters defined for "CW Operation" are ignored.

Number of pulses: Number of a series of successive pulses to be computed.

Q-switch induced loss during load phase: A high relative loss has to be defined representing the artificial loss generated by the closed Q-switch.

Pulse repetition frequency [kHz]: self-explanatory

Pulse repetition period [μs]: Distance between two pulse trigger events, i.e., reciprocal value of pulse repetition frequency.

Load period [μs]: During this period a high population inversion is generated.

Opening period (pulse build-up) [ns]: Time period elapsing until the Q-switch becomes transparent after being triggered.

Pulse period [ns]: This quantity does not represent the physical pulse width, but only defines a computational time domain used for the discretization of period IIa as introduced at the beginning of section 7.6. Dependent on the cavity configuration it may be necessary to make the pulse period much longer than the pulse width to avoid that the pulse extends into the relaxation period.

load period + opening period + pulse period must be smaller than the **pulse repetition period**. For computation of multiple pulses, the remaining time is the relaxation period defined as period IIIb at the beginning of section 7.6 Q-Switching Analysis. The relaxation period should be comparable to the pulse period or smaller, but is not critical. Start a trial run, open the diagram **Power output over time scale**, and zoom in a single pulse. If the pulse shape is truncated, increase the **pulse period**, otherwise it might be possible to shorten it, to save computational time.

Number of time steps during load period: Since no rapid changes take place during the load period, a moderate number of steps can be used.

Number of time steps during opening period: Since the opening period is short, a moderate number of steps is sufficient.

Number of time steps during pulse period: Since the pulse period involves high derivatives versus time, a large number of steps is necessary.

Number of time steps during relaxation period: Since the time steps during **relaxation period** can be much longer than during **pulse period**, usually a moderate number is sufficient. However, if the **pulse repetition frequency** is low and the **load period** is short, the **relaxation period**, which is automatically computed by the use of the above defined parameters, may become long. Since time steps > 2 ns may deliver erroneous results, it is recommended to increase the number of time steps in these case, and to compute one pulse only.

Mode of Pumping: This option allows for selecting the type of pumping.

- If **CW** is selected, it is assumed, that a cw pump beam is absorbed in the crystal, and the active Q-switch is opened with the frequency defined by **pulse repetition frequency**.
- If **Pulsed synchronous with Q-switch** is selected, it is assumed, that single pump pulses of constant intensity are applied extending over the load period. It is assumed, that the relaxation period between the load periods is long compared with the load period itself, and in consequence, that no interaction between the individual pulses takes place. Therefore, only one pulse is computed, and the relaxation period is neglected. It is recommended to make the **pulse period** long enough to take into account the decay of the pulse. For control you can open **2D - Power output over time** with the DMA Viewer. Nevertheless, it is necessary to define the **pulse repetition frequency**, since the power of the pump pulses is computed by dividing the time averaged pump power, which must be used for the current time-independent FEA, by the product **Pulse repetition frequency × Load period**. The accuracy of the time-independent approximation is sufficient, if the pump frequency is greater than 60 Hz. Time dependent thermal FEA is under development.

7.7.5 Tab "Apertures"

The apertures defined in the tab **Apertures** of the window **Parameter Field** of the LASCAD main program are not used with the DMA code. Position and type of aperture must be specially defined in this tab.

Resonator contains aperture: *True/False*

Type of aperture: *Rectangular/Elliptic/Gaussian*

Position of aperture along z-axis [mm]: Origin of the z-axis is the left mirror.

For rectangular or elliptic apertures:

Radius of aperture, x-dir. [mm]: self-explanatory

Radius of aperture, y-dir. [mm]: self-explanatory

For gaussian apertures:

1/e²-radius of gaussian transmission or reflection profile, x-dir. [mm]:

1/e²-radius of gaussian transmission or reflection profile, y-dir. [mm]:

The gaussian transmission related to the intensity is defined as

$$t(x, y) = \exp\left(-2x^2 / w_{trx}^2 - 2y^2 / w_{try}^2\right)$$

where $w_{trx}(y)$ is the 1/e²-radius of the profile. Concerning gaussian transmission profiles, see *A.E.Siegmann, LASERS, University Science Books, Chapt. 20.*

Gaussian apertures are not implemented yet in the LASCAD main program. To see the influence on the mode shape open

2D - Spot size in x(y)-dir of TEM00 along resonator axis with the DMA Viewer.

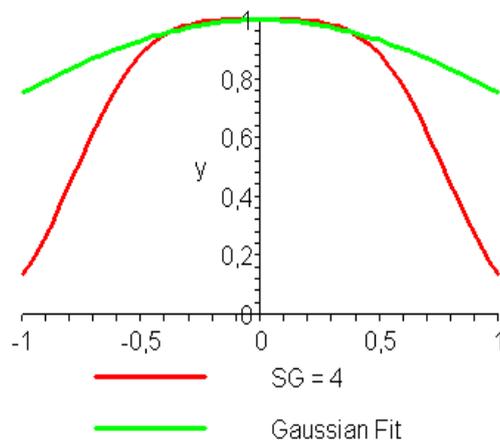
If an aperture coincides with the output mirror, it is assumed that its reflectivity vanishes outside the area defined by the aperture.

If the option **Gaussian** is selected, a mirror with the super-gaussian reflectivity

$$R(x, y) = R_0 \exp\left(-2\left|\frac{x}{w_{trx}}\right|^{SG} - 2\left|\frac{y}{w_{try}}\right|^{SG}\right) + R_{\min}$$

can be defined using the super-gaussian exponent $SG \geq 2$. R_{\min} is the **Peripheral bottom reflectivity**. For R_0 the program uses the value $R_0 = R_{\text{out}} - R_{\min}$ where R_{out} has to be defined as **Reflectivity of output mirror** in tab *Rate Equations* (Sec. 7.7.2).

Since super-gaussian apertures cannot be taken into account in the ABCD matrix algorithm, the inner part of the super-gaussian distribution is fitted by a gaussian approximation dependent on SG as shown in the diagram for $SG = 4$. This approximation seems to be appropriate, since only modes with a radius $< w_{trx}(y)$ deliver a significant contribution to overall mode shape. The obtained gaussian fit is used in the ABCD matrix algorithm.



7.7.6 Directory and File Management

The default directory for the DMA data is the subdirectory **DMA** of the LASCAD working directory. The DMA directory is used to save the parameter file of the DMA Code and for generated output. The default location can be changed by pressing *Change...* in the main DMA window. Note however, that changing the DMA directory is not permanent. A new DMA session will always start with the default directory.

7.7.6.1 Files in the DMA directory

- `dma.dat`
Contains the parameters for the DMA calculation. This file is generated each time the DMA calculation is run. The format of this file is subject to change.
- `dma.lcd`
A copy of the LASCAD project file containing the cavity configuration used for the DMA run.
- `output.txt`
A copy of the text output generated during the DMA calculation.
- `*.gnu`
2D plot data in Gnuplot (<http://www.gnuplot.info>) format (comments and pairs of xy-coordinates).
- `*.vtk`
3D plot data in VTK ASCII format.

8 Beam Propagation Method (BPM)

8.1 Window "Beam Propagation Method"

This window is activated by clicking "BPM/Run BPM" in the menu bar of the main window of "LASCAD". It is used to create the input data file for the beam propagation code and to start this code. The tab control in the upper part of the window has two tabs "General" and "Individual Propagation Steps".

Tab "General":

The first row contains a box showing the "Half width of the computational window". This parameter defines the half width of a quadratic channel along the resonator axis that is used for the computation. The cross section of this channel is subdivided into a grid with 128×128 equidistant points, which are used for the FFT.

The second row contains a box showing "The general width of propagation step". This parameter defines the grid points along the propagation axis where the complex amplitude of the wave front is being recalculated.

The frame below contains two check boxes.

If the upper box is checked the half width of the computational window is computed automatically using the maximum spot size along the propagation path which is multiplied by a factor 2. This turned out to be optimum setting if computation of the fundamental mode is intended.

If the lower box is checked the general width of propagation step is computed from gaussian analysis by multiplying the minimum spot size along the propagation path by a factor 3, which turned out to deliver reliable results within acceptable computational times.

If the upper or lower box is unchecked the related quantity can be defined arbitrarily. For instance, if you intend to include higher order transverse modes in the computation, it is recommended to increase the width of the computational window.

If upper or lower box are checked the related parameter corresponding to gaussian analysis is recomputed every time when a computation of the gaussian mode has been carried through.

The value entered for the general width of the propagation step defines a step which is approximately identical for all fields between the face elements shown in the mode plot window. Only for the thermally loaded crystal the step is equal to the z-axis grid size used by the FEA and is not smaller than the length of the crystal divided by 10.

Tab "Individual Propagation Steps":

To adjust the propagation step individually open the tab "Individual Propagation Steps". Here you can enter individual step sizes for the different fields. For the thermally loaded crystal the step cannot be made larger than the z-axis grid size used by the FEA, and not larger than $(\text{length of crystal})/20$. In addition, the propagation step generally cannot be made larger than $(\text{distance between elements})/10$. The individual

values are overwritten every time you change the **"General width of propagation step"** in tab "General".

The number box in the first row below the tab control defines the number of cavity iterations NCAV that means full round trips of the wave front in the resonator.

The up-down control in the next row allows selecting the number of samples of the 2D transverse beam profile which are used for the numerical eigenmode analysis. The samples are taken at the right end mirror. The number of samples must be a power of 2 since a FFT technique is used for this analysis as described in section 4.3. NCAV must be greater than the number of samples; otherwise eigenmode analysis cannot be accomplished. In order to meet this condition NCAV is adjusted automatically every time you change the number of samples.

The number box in the third row defines the frequency of dumping intensity and phase profile at the right mirror into files. The default value 30 means that this data are saved each time after 30 cavity iterations to `File19.dat` (intensity) and `File08.dat` (phase). At the end of the computation these data are copied to the files `X001.dat` and `Y001.dat` which can be used to show the beam profile as described in section 8.4.

Check the box **"Compute eigenmodes"** to compute eigenmodes and display them at the end of the BPM run.

Check the box **"Use dynamic gain model to compute laser power output"**, if an active crystal is in the cavity. With this setting a dynamic gain model based on the laser rate equations is used to compute laser power output and beam quality. Both parameters are written to the output file `lyra.txt` in the subdirectory BPM of the working directory after every cavity iteration. The laser power output is additionally shown in the window **"Beam Radius and Laser Power versus Cavity Iteration"** during the run of the BPM code, see Sect. 8.2. Uncheck this box, if the dynamic gain model shall not be used.

Frame **"Use of Apertures"**:

This frame contains 3 option buttons to control use of apertures with the BPM code.

If **"Use twice the spot size of the Gaussian fundamental at the end mirrors"** is checked, apertures only are used at the end mirrors. The radii of these apertures are equal to the corresponding gaussian spot sizes times 2. This setting turned out to deliver best convergence to the fundamental mode. Notice, that this option is checked by default, when the BPM input window is opened.

If **"Use half width of computational window"** is checked, the beam propagates within the full quadratic area of computational window and is not confined by individual apertures.

If **"Use aperture settings"** is checked, the apertures defined in the index card with tab "Apertures" of the window "Parameter Field" are used.

Frame "Initial Beam Profile":

This frame contains 3 option buttons to control the initial beam profile.

If "**Gaussian with spot size obtained from ABCD code**" is checked, the computation starts with the profile of the Gaussian fundamental at the left mirror obtained by ABCD matrix analysis. Also the curvature of the phase front is taken into account.

If "**Gaussian with spot size defined in the box below**" is checked, the computation starts with a Gaussian profile with spot size (= radius where the intensity is dropping to $1/e^2$) defined in the box at the bottom of the frame. The curvature of the phase front at the left mirror is taken into account.

If "**Circular top hat with the radius defined in the box below**" is checked, the computation starts with a circular top hat profile with plain phase front.

The button "**Run BPM**" creates the files Input_basic.txt and Input_wgpc.txt, which are used as input for the BPM code. Before these files are created a window is popping up asking, if you would like to keep already existing files. The window of the BPM code, which is starting now, has the caption "**BPM**". It has its own menu and several child windows, which are explained in section 8.7. All files created by the BPM code are written to the subdirectory "**BPM**" of the working directory.

In case of a ring resonator an output mirror must be defined which can be any mirror between the leftmost and rightmost element shown in the mode plot. The element number of this mirror has to be entered in the box in the bottom row of the tab element. The rightmost element cannot be selected as output mirror. In case of a ring resonator, it is assumed that the leftmost element is a dummy element which is at the same position as the rightmost element. The rightmost element is assumed to be the input mirror, and the beam starting at the left side of the mode plot is assumed to start immediately after this input mirror. If only ABCD matrix analysis is being carried through for a ring cavity the leftmost element must not necessarily be a mirror, also it is not necessary to define an output mirror.

8.2 Window "Beam Radius and Laser Power versus Cavity Iteration"

This window shows, how beam radius and laser power output develop with increasing number of cavity iterations. It is opened automatically when the BPM starts to compute cavity iterations, it can however be opened all time by clicking "**BPM/Show Beam Radius**" in the menu bar of the main window of LASCAD. Originally the BPM code computes the averaged $1/e$ -radius of the intensity profile at the right mirror. The computation of the $1/e$ -radius was preferred to the computation of the $1/e^2$ -radius since the bottom of the profile sometimes is a bit noisy, and therefore the $1/e$ -radius is more meaningful. However, in order to facilitate comparison of the results for the radius with the gaussian spot size, which by general agreement corresponds to the $1/e^2$ -radius of the intensity, the radius shown in the diagram is multiplied with the square root of 2 by default, when the window is opened. You can however show the

originally computed 1/e-radius by checking the option button "**radius [1/e]**" at the lower left corner of the window.

Type desired values into the boxes labeled with "**Xrange**" and "**Yrange**", to change the horizontal or the vertical scale of the diagram, respectively.

Check the box "**Show Laser Power Output**" to show laser power output together with beam radius versus cavity iteration. Power output only can be shown if the box "**Use dynamic gain model to compute laser power output**" is checked in the window "**Beam Propagation Method**" before running the BPM code.

8.3 Window "Beam Quality versus Cavity Iteration"

This window shows, how the beam quality parameters M_x^2 and M_y^2 develop with increasing number of cavity iterations. It can be opened by clicking "**BPM/Show Beam Quality**" in the menu bar of the main window of LASCAD.

To change the horizontal scale of the diagram, type the desired value into the box labeled with "**Xrange**".

8.4 Window "Intensity and Phase at Right End Mirror"

Clicking "BPM/Show Beam Profile" in the menu bar of the main window „LASCAD“ opens the window "Intensity and Phase at Right End Mirror". It is used to show intensity and phase distribution at the right end mirror as obtained during and at the end of the iteration process. The drop-down list box at the lower right corner allows selecting the cavity iteration for which the profile shall be shown. Click a number in the list and the intensity profile belonging to this cavity iteration will be shown. The numbers shown in the list depend on the number which you have entered into the box "**Frequency of saving intensity + phase profile**" in the window "**Beam Propagation Method**" as described in section 8.1. To show the phase distribution check the option "**Phase**" in the frame. Checking the related option buttons allows showing the profile as a solidly filled surface or as a deformed grid structure. With the scroll bars at right and bottom border of the graph you can modify the viewing angle. With the horizontal bar you can change the rotation angle about the vertical axis of the graph. With the vertical bar you can change the elevation angle of the graph. The "**Print**" button can be used to print the graphs. To copy the graphs to a file or to the clipboard use the items "**Print to File**" or "**Copy to Clipboard**" in the menu of the main window "LASCAD".

8.5 Window "Spectrum of Eigenfrequencies"

Clicking "BPM/Show Frequency Spectrum" in the menu bar of the main window LASCAD opens the window "Spectrum of Eigenfrequencies" that shows the result of the eigenfrequency analysis. To change the horizontal scale of the plot enter an other value into the box "**Frequency Range**" at the lower left corner and then press the ENTER key or click "**Draw**". The accuracy and resolution of the spectrum depends on the number of samples taken for this analysis that can be defined in the window "**Beam Propagation Method**" as described in section 8.1. Unfortunately, the method currently used for frequency analysis seems not to deliver very accurate results for resonators

with strongly focusing elements or long resonators. A more powerful algorithm is under development.

8.6 Window "Eigenmodes"

Clicking "BPM/Show Eigenmodes" in the menu bar of the LASCAD main window opens the window "Eigenmodes", which is used to show the intensity profile of the eigenmodes. The modes belonging to the first seven strongest peaks of the positive branch of the frequency spectrum are shown. Clicking a frequency value in the list of the drop-down box at the lower right corner selects an eigenfrequency for which the profile is shown. The next box to the left gives the $1/e^2$ -radius of the profile. The buttons in the frame allow selecting to plot the intensity or the real part of the complex amplitude. Again you have the options to show the profile as a solidly filled surface or as a deformed grid structure. The scroll bars at right and bottom border of the graph can be used to modify the viewing angle as described in section 8.4.

8.7 Windows of the BPM Code

The main window of the BPM code has the title "**BPM**" and its own menu bar. The most important menu items and child windows are:

"**File/Exit**" stops the code immediately

"**File/Soft_Exit**" terminates the code after finishing the current round trip and writes the data to different files.

"**Action/New_Input**" allows entering new data for cavity iteration and dumping frequencies for data. See the items described in sec. 8.1.

"**Output/PlotBeamRadius**" opens the child window "**1/e radius**", which shows the course of the $1/e$ beam radius with increasing cavity iteration similar to the LASCAD window "Convergence of Beam Radius with Cavity Iteration".

The child window "**BPM Output**" is opened automatically at the start of the iteration process, and records the progress of the Fox and Li type iteration.

The child window "**Interpolation of FEA Results**" is opened automatically when the interpolation of the FEA results starts, and records the progress of interpolation.

After the start of the Fox and Li type iteration a 2D Array Viewer with title "Intensity at Right (End) Mirror" is opened which shows the intensity profile at the right end mirror. This window has its own menu and tool bar, which allow modifying some features like background color etc. The plot is created by the use of OpenGL, and can be rotated with the mouse. It is being refreshed after each cavity iteration. In addition you can use the BPM menu item "View/Intensity along beam path" to show the intensity distribution along resonator axis and x-axis.

A detailed listing of numerical results created by the BPM code such as laser power output, beam quality etc. can be found in the file `lyra.txt` in the subdirectory BPM of the LASCAD working directory. A user guide for the BPM code can be found in the file `BPM_user_guide.chm` in the subdirectory Documentation of the LASCAD application directory.

9 Restrictions of the Demo

The wavelength is fixed to 1.134 μm

- Only 4 face elements are possible
- For standing wave resonators the values of type parameters of face elements 0 and 3 are fixed and cannot be changed.
- For ring resonators and external beam the value of the type parameter of face element 1 is fixed and cannot be changed.
- The latter also holds when elements are inserted or cleared, which leads to a somewhat different behavior compared with the full version.
- For the pump power only the values 20, 90 and 360 W are possible. Any other value being entered is replaced by one of these default values dependent on its size.
- The simulation of misalignment effects is not available with the demo. This feature is presently being tested and will be available with the next full version of LASCAD.
- For the side pumped configurations the diameter of the rod is fixed to 3 mm, for all other configurations diameter or width of rod are restricted to 2 mm.

Appendix A Computation of the Absorption Coefficient

After entrance into the crystal the pump beam is absorbed by the lasing atoms. Therefore, its intensity decreases exponentially according to

$$I(z) = I_0 \exp(-\alpha z), \quad (\text{A1})$$

where z is the distance from the surface and α is the absorption coefficient. The latter one is proportional to the concentration of the lasing atoms, i. e. the doping level. Typical absorption coefficients for common laser materials Nd:YAG, Yb:YAG, etc. and laser diodes can be found in the book *Principles of Lasers* of Orazio Svelto, Chapt. 6 (Plenum Press, N.Y., 1998).

To compute the absorption coefficient it is necessary to have a closer look on the physical details. Physically, pump light as well as absorption coefficient show a spectral distribution. Examples are shown in Figs. 1 to 3: Fig. 1 shows the absorption spectrum for 1% Nd:YAG, Fig. 2 shows the absorption spectrum for 1% Nd:YVO₄, Fig. 3 shows the emission spectrum of a P1202 laser diode of Coherent, Inc. for different diode currents. Emission spectra usually are found on the data sheet of the diode. Due to the spectral dependency Eq. (A1) turns out to be only an approximation, the correct expression is

$$I(z) = \int_{\lambda_1}^{\lambda_2} f_e(\lambda) \exp(-\alpha(\lambda) z) d\lambda \quad (\text{A2})$$

where λ_1 and λ_2 are the lower and upper limit, respectively, of the emission spectrum $f_e(\lambda)$ of the diode and $\alpha(\lambda)$ is the absorption coefficient. Eq.(A2) can be evaluated at different degrees of approximation. Simplest method is to carry through an average of the absorption spectrum around the peak wave length of the diode. For a more sophisticated evaluation, it is necessary to compute $I(z_n)$ for different distances z_n by numerical integration. The obtained data set $z_n, I(z_n)$ then can be fitted by an exponential function to obtain an averaged absorption coefficient α . Of course, α also can be determined experimentally by measurement of the intensity of the diode light remaining behind crystal layers of different thicknesses.

If the absorption spectrum depends on the polarization of the pump light as shown in Fig.2, Eq.(A2) has to be replaced by

$$I(z) = \int_{\lambda_1}^{\lambda_2} f_e(\lambda) [\rho_a \exp(-\alpha_a(\lambda) z) + \rho_c \exp(-\alpha_c(\lambda) z)] d\lambda \quad (\text{A3})$$

where $\alpha_a(\lambda)$ and $\alpha_c(\lambda)$ are the absorption coefficients for light polarized parallel to axis a and c, respectively. ρ_a and ρ_c are the relative contributions to the pump spectrum polarized parallel to axis a and c, respectively. Eq. (A3) can be fitted by the expression

$$I(z) = I_0 [\rho_a \exp(-\alpha_a z) + \rho_c \exp(-\alpha_c z)]. \quad (\text{A4})$$

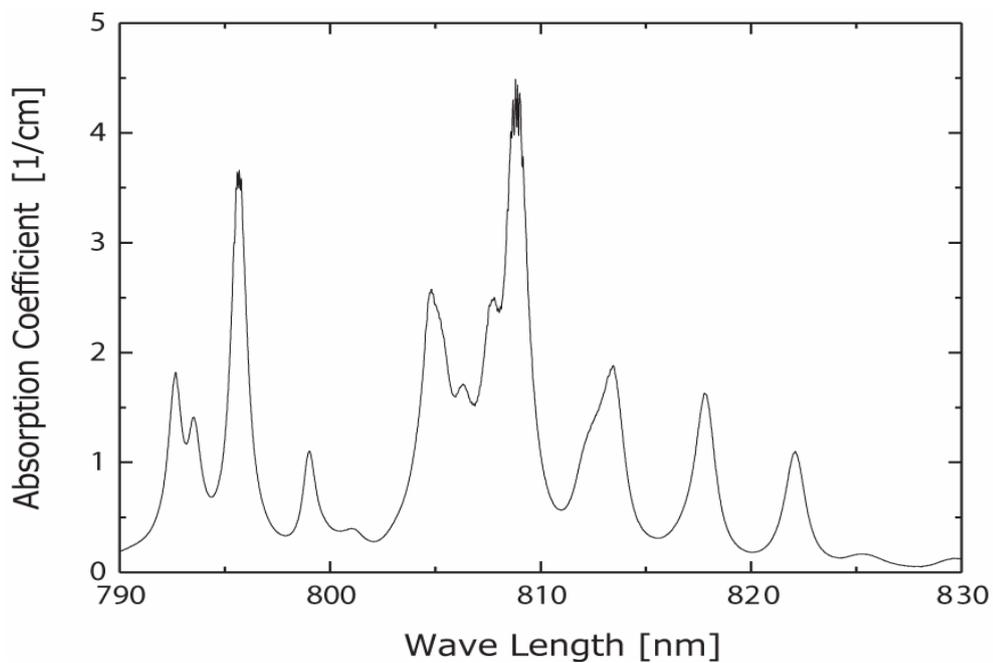
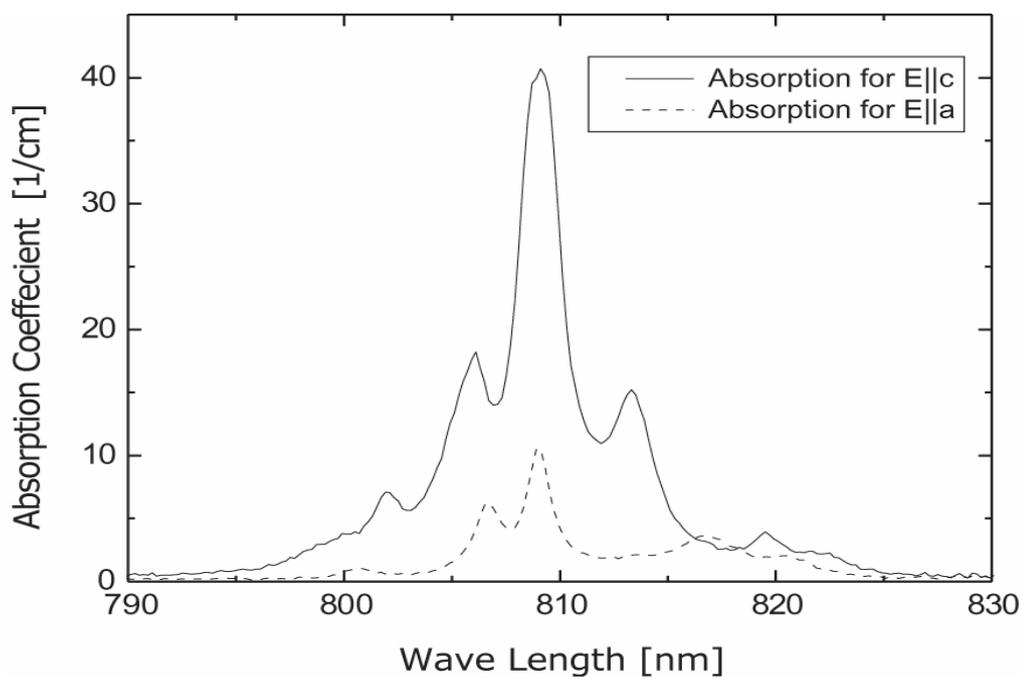


Fig. A1 Absorption spectrum of 1 atomic % Nd:YAG

Fig. A2 Absorption spectrum of 1 atomic % Nd:YVO₄

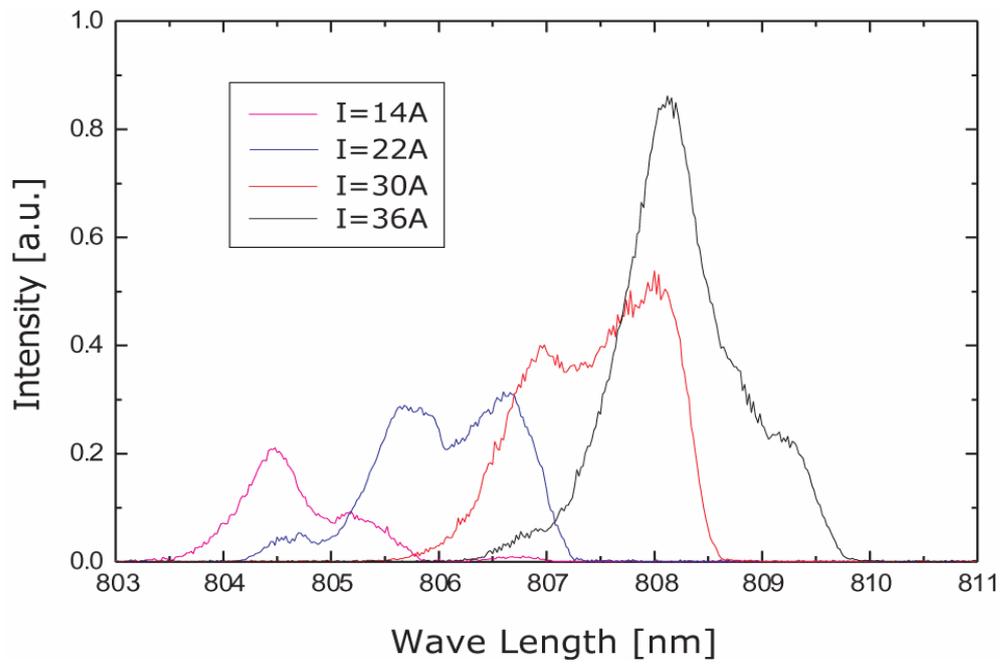


Fig. A3 Emission spectra of high power laser diode P1202 of Coherent, Inc. for different values of diode current at constant temperature 20° C.