Computation of
Laser Power Output
for CW Operation

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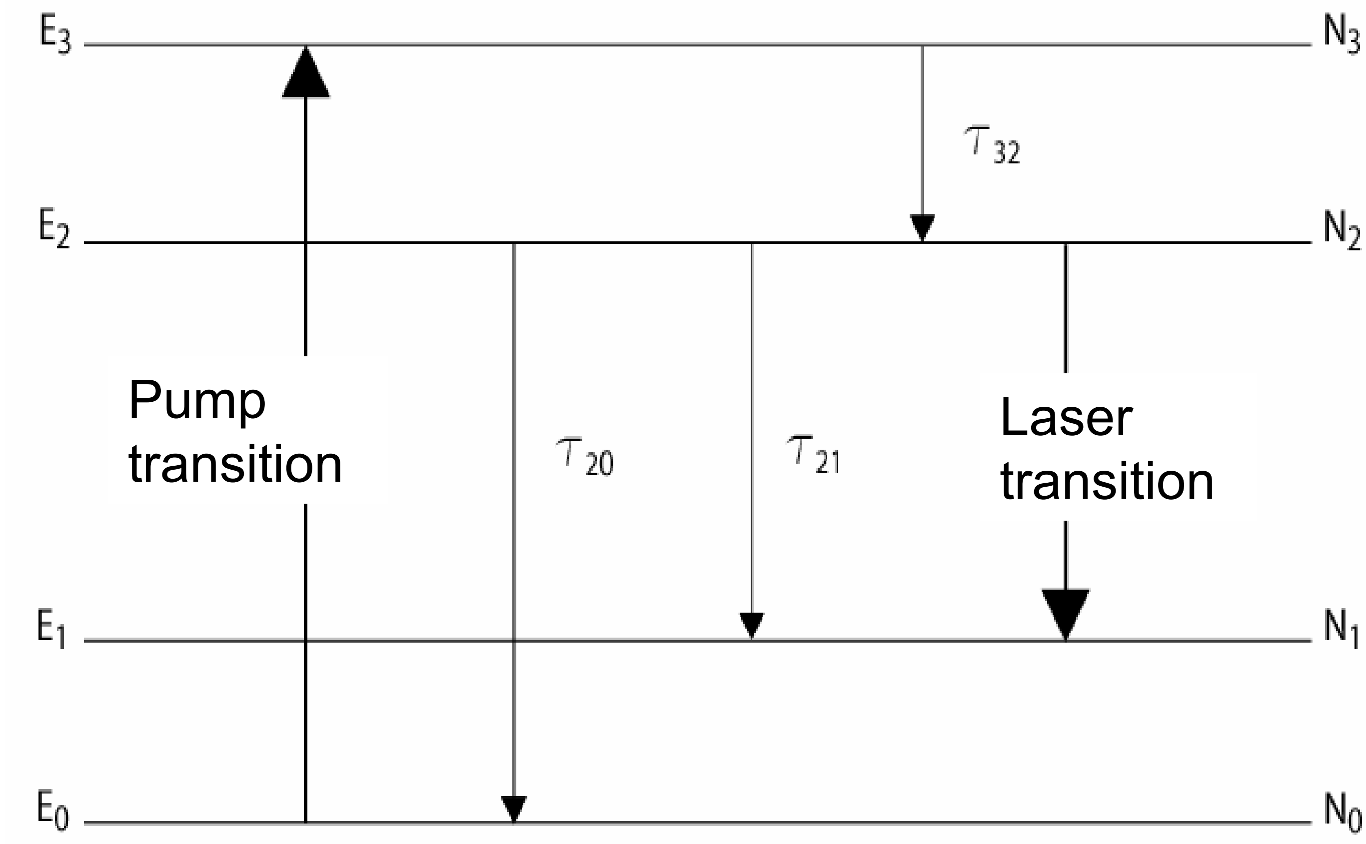


Energy levels, population numbers, and transitions for a 4-level laser system

Rate Equations for a 4-Level System

$$\frac{\partial N}{\partial t} = R_p - WN - \frac{N}{\tau}\ , \qquad \frac{dS_L}{dt} = \iiint_a W\, N\, dV - \frac{S_L}{\tau_c}$$

$N(x,y,z) = N_2 - N_1$	population inversion density (N1 ~ 0)
R_p	pump rate
$W(x,y,z)$	transition rate due to stimulated emission
τ	spontaneous fluorescence life time of upper laser level
S_L	number of laser photons in the cavity
τ_C	mean life time of laser photons in the cavity

The **pump rate** is given by

$$R_p = \eta_p S_p p_0$$

η_P	pump efficiency
$p_0(x,y,z)$	absorbed pump power density distribution normalized over the crystal volume
S_p	total number of pump photons absorbed in the crystal per unit of time

The **transition rate due to stimulated emission** is given by

$$W = \frac{c\sigma}{n} S_L s_0(x, y, z)$$

σ stimulated emission cross section

n refractive index of laser material

$s_0(x,y,z)$ normalized distribution of the laser photons

Detailed Rate Equations of a 4-Level Systems

$$\frac{\partial N}{\partial t} = R_p - N\frac{c\sigma}{n}S_L\, s_0(x,y,z) - \frac{N}{\tau}$$

$$\frac{dS}{dt} = S_L\left[\iiint_a \frac{c\sigma}{n} N\, s_0(x,y,z)\, dV - \frac{1}{\tau_c}\right]$$

Condition for equilibrium

$$\partial N/\partial t = dS/dt = 0$$

Using the equilibrium conditions, and carrying through some transformations one is getting a recursion relation for the number of laser photons in the cavity

$$S_L = \tau_c \, \eta_p \, S_P \iiint_a \frac{p_0(x,y,z)}{1 + \dfrac{n}{c\sigma\tau \, S_L \, s_0(x,y,z)}} dV$$

This equation can be solved by iterative integration. The integral extends over the volume of the active medium. The iteration converges very fast, as starting condition

$$S_L = \tau_c \, \eta_p \, S_P$$

can be used.

The laser power output is obtained by computing the number of photons passing the output coupler per time unit. This delivers for the power ouput the relation

$$P_{out} = h\nu_L \, S_L \, \frac{c(-\ln(R_{out}))}{2\widetilde{L}}$$

R_{out}	reflectivity of output mirror
c	vacuum speed of light
ν_L	frequency of laser light
h	Planck's constant

$\widetilde{L} = L_r + (n-1)L_a$ optical path length

To compute τ_C we divide the time t for one round-trip

$$t = \frac{2\,\tilde{L}}{c}$$

by the total loss T_T during one round-trip

$$T_T = L_{roundtrip} - \ln(R_{out})$$

Here $L_{roundtrip}$ represents all losses during one roundtrip additional to the loss at the output coupler. Using the above expression one obtains

$$\tau_c = \frac{2\tilde{L}}{c(L_{roundtrip} - \ln(R_{out}))}$$

Using the above relations one obtains for the laser power output the recursion relation

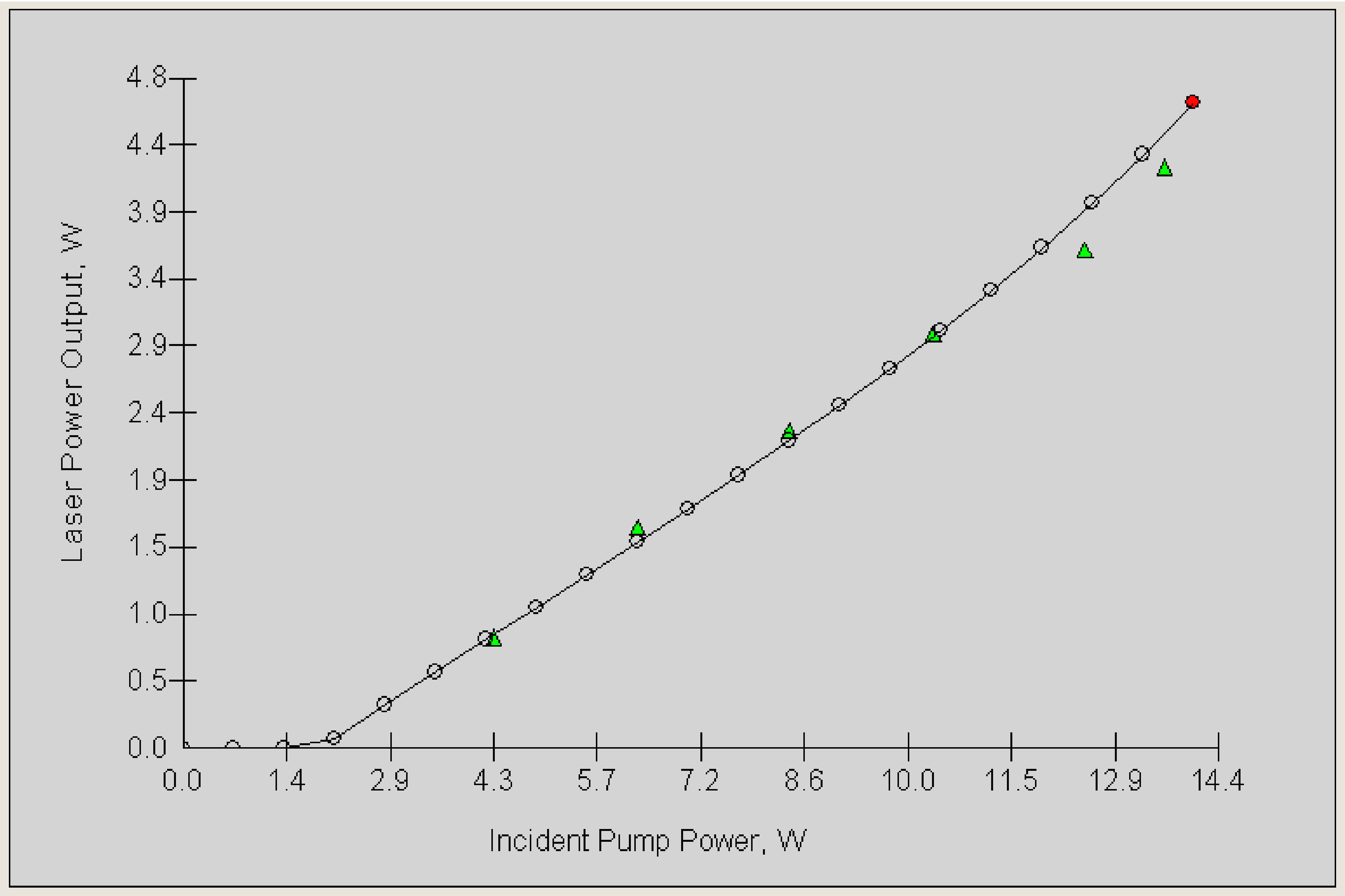
$$P_{out} = \eta_p P_P \frac{h\nu_L}{h\nu_P} \frac{-\ln(R_{out})}{T_T} \iiint_a \frac{p_0}{1 + \frac{n\, h\nu_L\, c\, T_M}{2 P_{out}\, \tilde{L}\, s_0\, c\, \sigma\, \tau}} dV$$

Here

$$P_P = h\nu_P S_P$$

is the totally absorbed pump power per time unit, ν_P is the frequency of the pump light

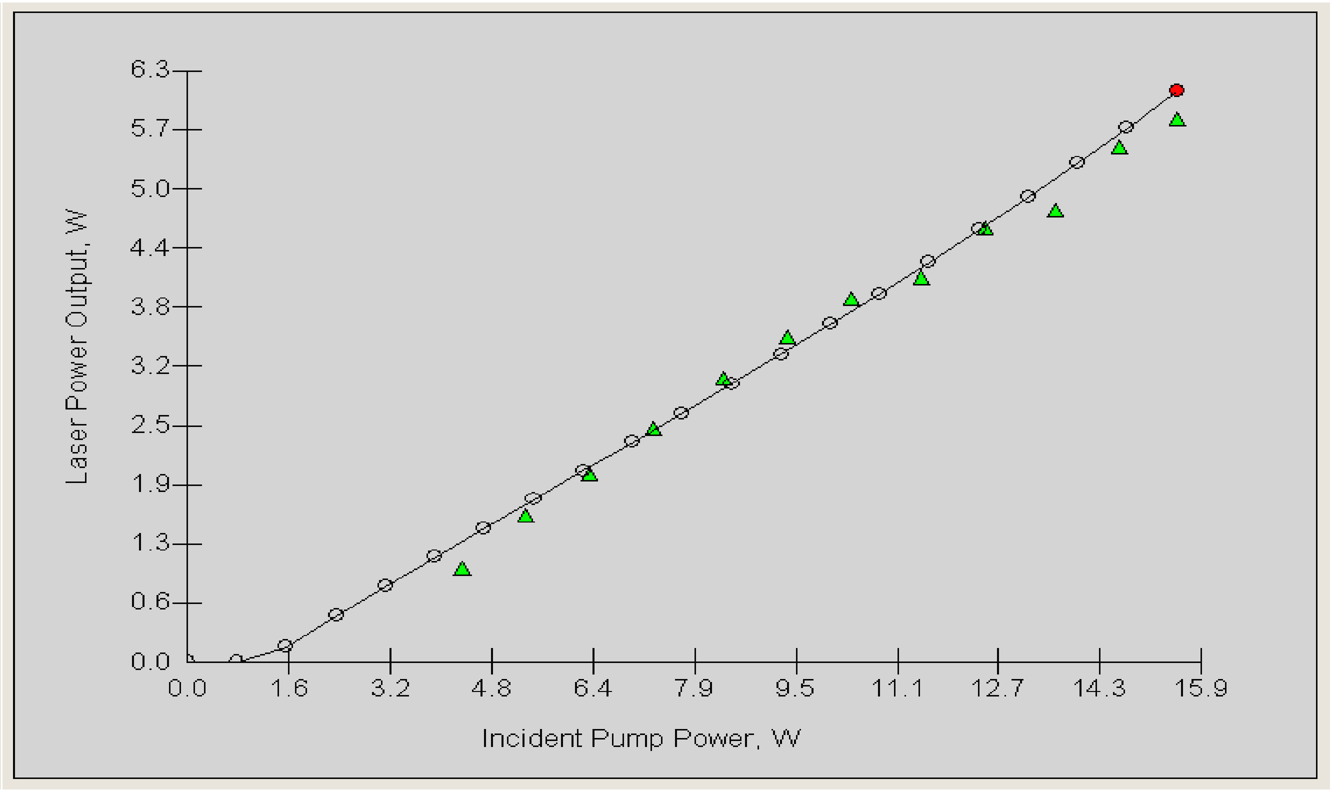
The next viewgraphs are showing results of comparison between experimental measurements and simulation for Nd:YAG and Nd:YVO4. The agreement between the results turned out to be very good.



Power output vs. pump power for 1.1 at.% Nd:YAG

▲ Measurement

o—o Computation

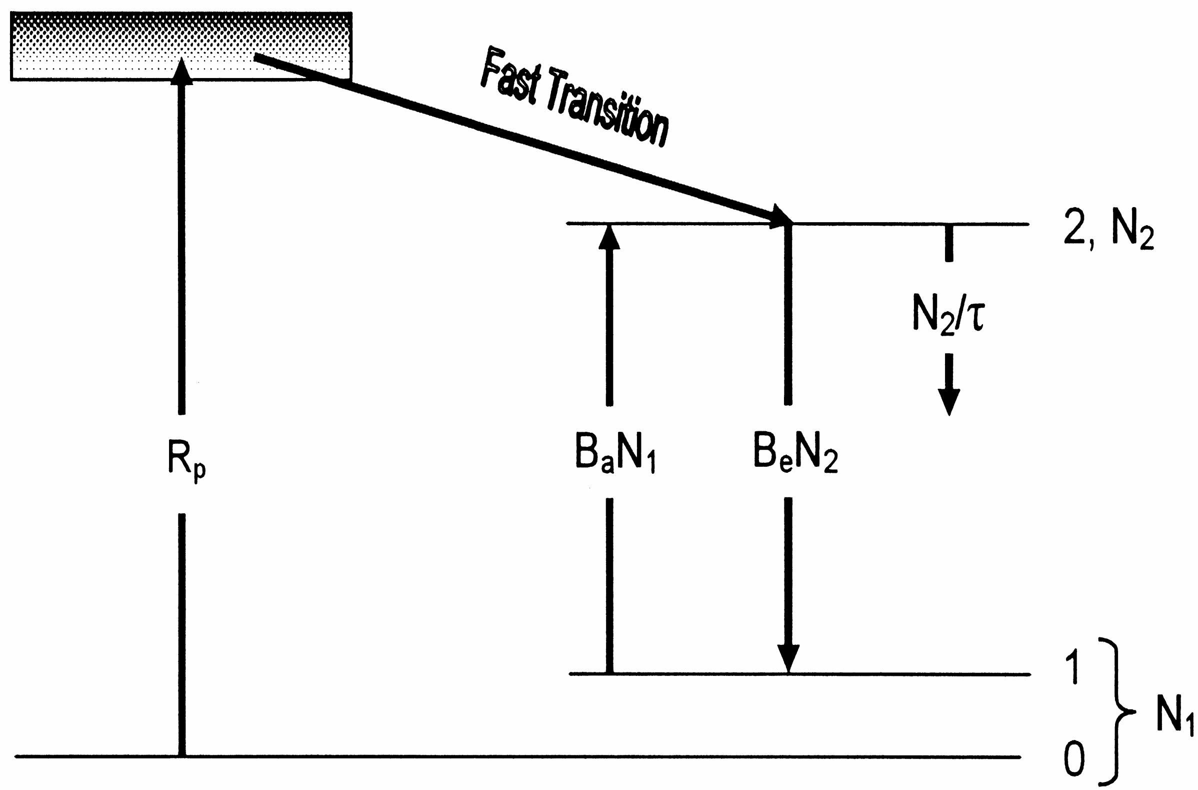


Power output vs. pump power for 0.27 at.% $Nd:YVO_4$

△ Measurement

o—o Computation

In similar way the laser power output for a quasi-3-level laser system can be computed



Energy levels, population numbers, and transitions for a quasi-3-level laser system

Rate Equations for a Quasi-3-Level System

$$N_t = N_1 + N_2$$

$$\frac{\partial N_2}{\partial t} = R_p - B_e N_2 + B_a N_1 - \frac{N_2}{\tau}$$

$$\frac{\partial S_L}{\partial t} = \iiint_a \left(B_e N_2 - B_a N_1\right) dV - \frac{S_L}{\tau_C}$$

N_t doping density per unit volume

B_e transition rate for stimulated emission

$$B_e = \frac{c\sigma_e}{n} S_L s_0(x, y, z)$$

B_a transition rate for reabsorption

$$B_a = \frac{c\sigma_a}{n} S_L s_0(x, y, z)$$

$\sigma_e(T(x,y,z))$ effective cross section of stimulated emission

σ_a effective cross section of reabsorption

c the vacuum speed of light

To solve the rate equation again equilibrium conditions are used

$$\partial N/\partial t = dS/dt = 0$$

After some transformations this recursion relation is obtained

$$P_{out} = \frac{h\,c\,T_M}{\lambda_L\,T_T} \iiint_a \frac{q_\sigma\,\eta_p \lambda_p\,P_p\,p_0/(h\,c) - (q_\sigma - 1)\,N_t/\tau}{q_\sigma + \dfrac{h\,c\,T_M}{P_{out}\,(s_{GR} + s_{GL})\,\sigma\,\tau\,\lambda_L}}\,dV$$

This recursion relation differs from the relation for 4-level-systems only due to the term

$$q_\sigma = 1 + \frac{\sigma_a}{\sigma_e}$$

$$\sigma_a \to 0 \quad \Rightarrow \quad q_\sigma \to 1$$

For $q_\sigma = 1$ the above relation goes over into the relation for 4-level systems

The parameter q_σ depends on temperature distribution due to temperature dependence of the cross section σ_e of stimulated emission. σ_e can be computed by the use of the method of reciprocity. As shown in the paper of Laura L. DeLoach et al. , IEEE J. of Q. El. **29**, 1179 (1993) the following relation can be deducted

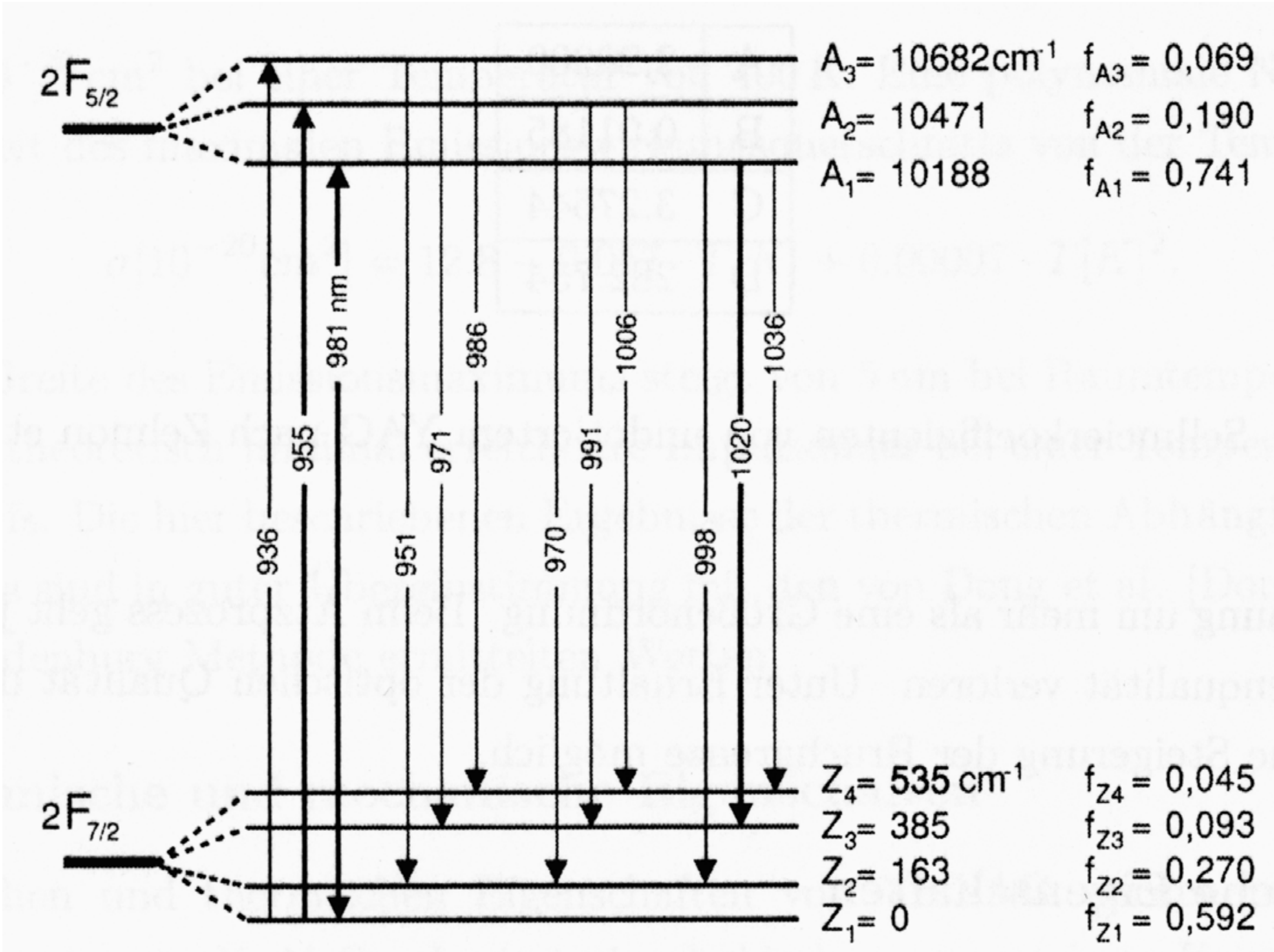
$$\sigma_e = \sigma_a \frac{Z_l(T(x,y,z))}{Z_u(T(x,y,z))} \exp\left(\frac{E_{ZL} - h\nu)}{kT(x,y,z)} \right)$$

Z_u and Z_l are the partition functions of the upper and lower crystal field states

EZL is the energy separation between lowest components of the upper and the lower crystal field states.

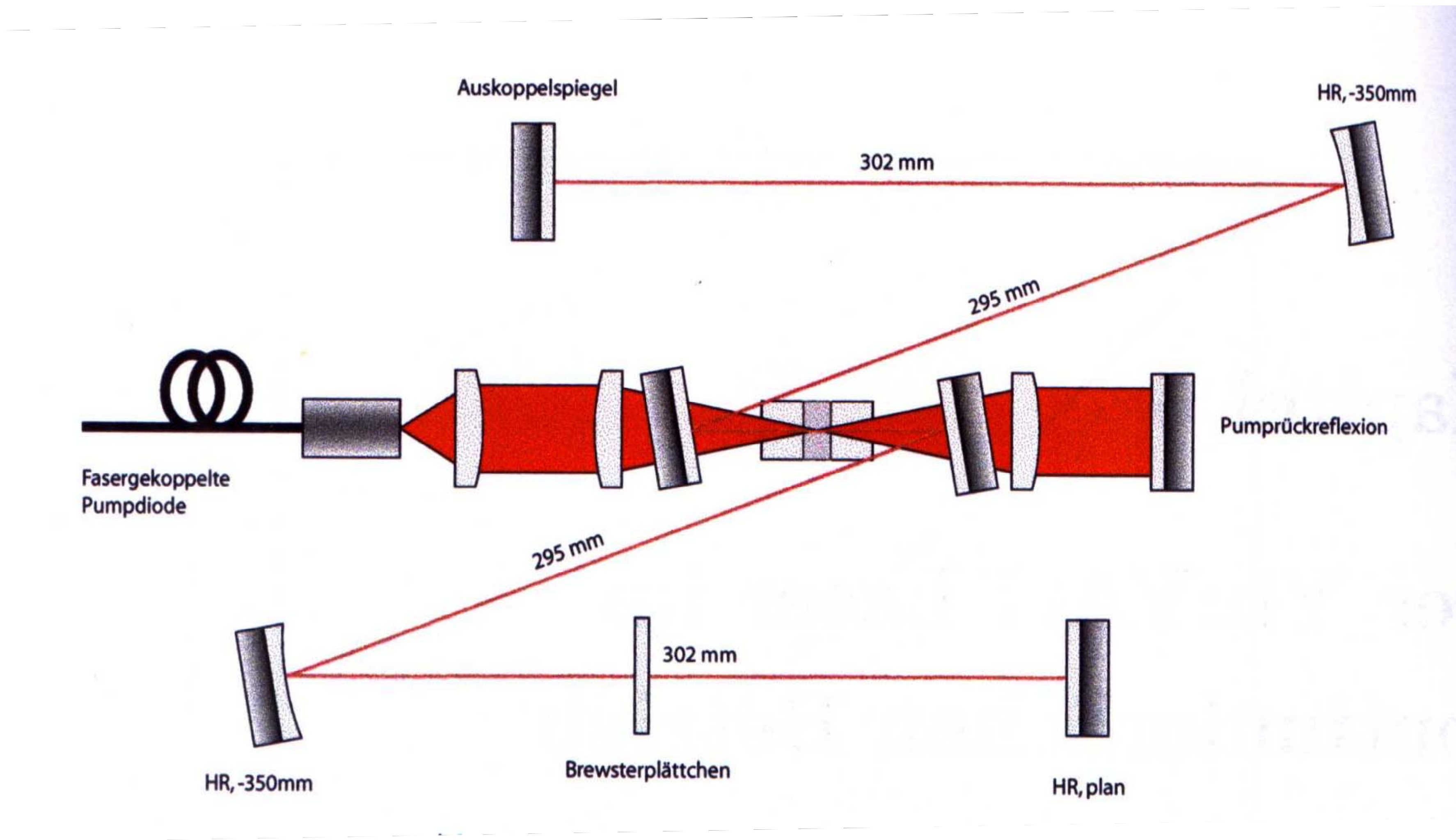
k is Boltzmann's constant

T(x,y,z) [K] is the temperature distribution in the crystal as obtained from FEA.

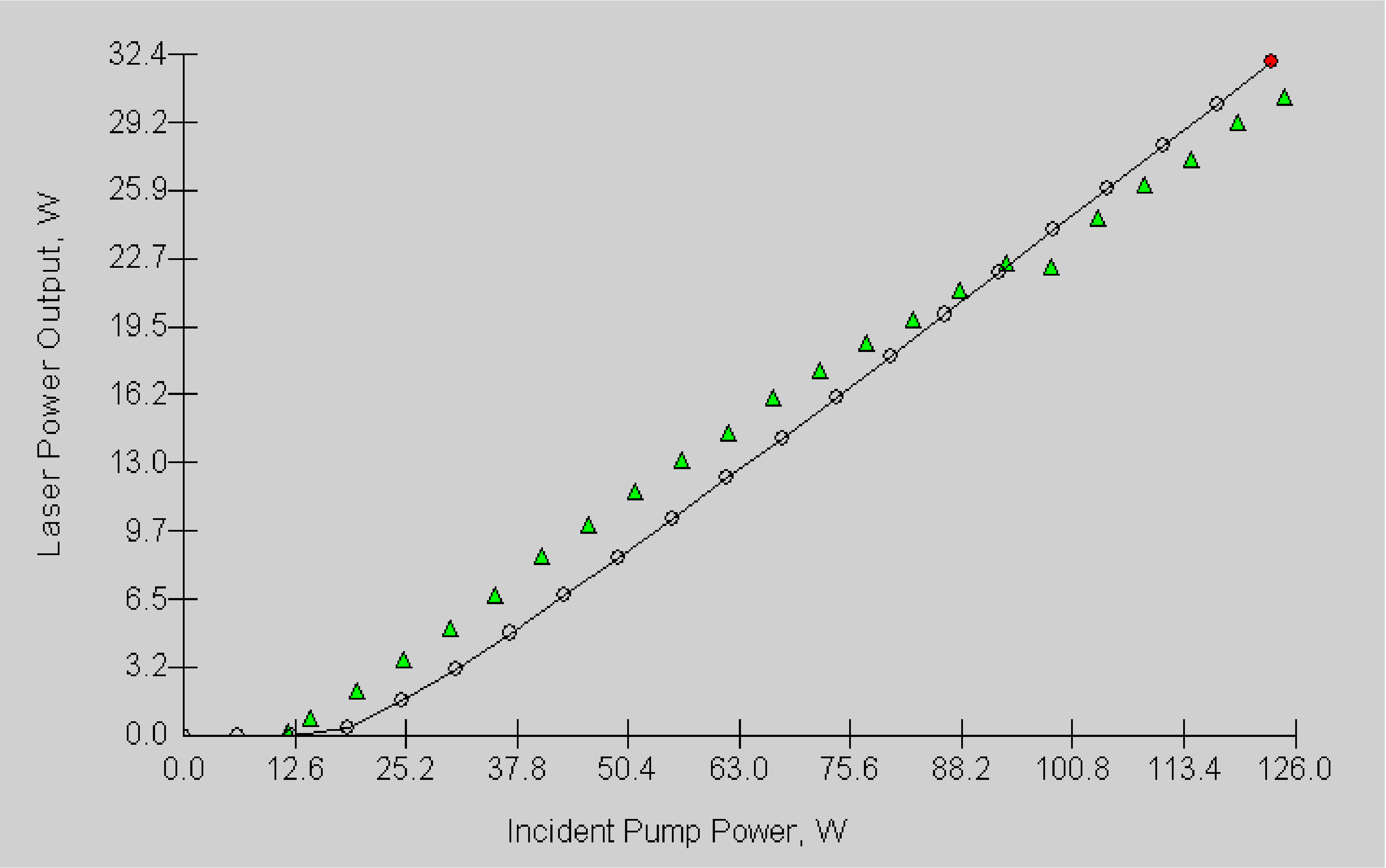


Energy levels and transitions for the

Quasi-3-Level-Material Yb:YAG



Yb:YAG cw-Laser, Laser Group Univ. Kaiserslautern



Output vs. Input Power for a 5 at. % Yb:YAG Laser

▲ Measurements: Laser Group, Univ. Kaiserlautern

o—o Computation Using Temperature Dep. Stim. Em. Cross Section