

LASCAD Tutorial No. 4:
Dynamic analysis of multimode competition and Q-Switched operation

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Note:

The numerical results shown in this tutorial, such as in Tab. 1, 2 and 3 are obtained, if for the computation the full version of the LASCAD program is used. If the demo version is used the results are slightly different, since the wavelength is fixed to 1.134 nm which differs from the correct wavelength of a Nd:YAG laser.

1 Introduction

The purpose of the Dynamic Multimode Analysis (DMA) is to analyze multimode and Q-switch operation of lasers. To this end the transverse mode structure in the cavity is approximated by a set of M Hermite-Gaussian (HG) or Laguerre-Gaussian (LG) modes. Since HG and LG modes represent sets of orthogonal eigenfunctions with different eigenfrequencies, it is assumed that each transverse mode oscillates independently, and therefore the influence of short-time interferences between the modes can be neglected on average. Based on this assumption, the interaction between the population inversion density and the photon numbers in the oscillating modes is described by the following set of time dependent 3D rate equations

$$S_C(t) = \sum_{i=1}^M S_i(t) \quad i=1, \dots, M, \quad (1)$$

$$\frac{\partial S_i}{\partial t} = \frac{c\sigma}{n_A} \int_{\Omega_A} N S_i s_i dV - \frac{S_i}{\tau_C}, \quad (2)$$

$$\frac{\partial N}{\partial t} = -\frac{c\sigma}{n_A} N S_C s_C - \frac{N}{\tau_f} + R_p \frac{N_{dop} - N}{N_{dop}} \quad (3)$$

Eqs.(1 to 3) describe the interaction between the population inversion density $N(x,y,z,t)$, the total photon number $S_C(t)$ in the cavity with associated normalized photon density distribution $s_C(x,y,z,t)$, and the photon numbers $S_i(t)$ belonging to the individual transverse modes with associated normalized photon density distributions $s_i(x,y,z,t)$ of the individual modes. The obtained time dependent photon numbers $S_i(t)$ in the individual modes and the photon density distributions $s_i(x,y,z,t)$ are used to describe their contribution to the whole transverse mode structure. In this way, a time dependent analysis of mode competition effects and multimode behavior of the cavity is obtained. This result is used to compute beam quality and laser power output, and to describe time dependence of Q-switch operation.

In Eqs.(2 and 3) the following parameters are used:

n_A	refractive index of the active medium,
c	vacuum speed of light,
$N(x,y,z,t) = N_2 - N_1$	population inversion density ($N_1 \sim 0$),
$R_p = \eta_p P_a / h \nu_p$	pump rate,
η_p	pump efficiency,
$P_a(x,y,z)$	absorbed pump power density,
σ	effective cross section of stimulated emission,
τ_C	mean life time of laser photons in the cavity,
τ_f	spontaneous fluorescence life time of upper laser level,
N_{dop}	doping density.

Eqs. (2 and 3) refer to a 4-level-system. Multimode analysis of quasi 3-level systems is under development. Fast decay rates between level 3 (pump level) and 2 (upper laser level), and between level 1 (lower laser level) and 0 are assumed.

Mathematical relations between the above parameters and the methods used to compute laser power output, and to model Q-switch operation and effects of apertures are described in detail in the manual, or can be shown by clicking the menu item "Help DMA Code" of the DMA activation window of LASCAD.

The following is a practical guide to the DMA Code. It explains how to define input values for the parameters shown in the individual tabs of the DMA GUI appropriately to model cw multimode operation, Q-switched operation, and the effect of apertures.

2 Modeling Laser power output, mode competition, and beam quality for cw operation

To work with the DMA code it is necessary to insert a thermally lensing crystal into the cavity. For instance follow the instructions in Tutorial No. 1 to prepare a model for an end pumped crystal. Alternatively, you can directly activate the cavity configuration according to Tutorial No. 1 by opening the project file Tutorial_1.lcd, which can be found in the subdirectory "Tutorials" of the LASCAD application directory. After you have executed the FEA code, and have inserted the crystal in the mode plot window, select **Dynamic Multimode Analysis** in the main LASCAD menu to open the DMA activation window. In this window click the button **Open GUI for DMA** to open the window **Dynamic Multimode Analysis**. This window has 5 tabs whose entries are described as follows.

2.1 Tab Gaussian Modes

Open this tab to select the **Type of gaussian modes** being used to approximate the laser mode structure. Select Hermite-Gaussian modes if the mode structure is astigmatic. For rotational symmetric cavity configurations it may be preferable to select Laguerre-Gaussian modes. However, if a high number of transverse modes has to be taken into account it is recommended to use Hermite-Gaussian modes. High order Laguerre-Gaussian modes are more complicated to handle numerically. Selection of mode type also can be carried through automatically by the program based on the astigmatism of the laser mode structure.

Maximum transverse mode order defines the highest transverse mode order N_{\max} taken into account in x- as well as in y-direction. However, since the total number M of modes increases according to $N_{\max}^2 + 2N_{\max} + 1$ the total number M can become very large with increasing N_{\max} so the computation takes a long time.

If a large N_{\max} is defined, it is necessary to make the **Number of grid points in x-and y-direction** large enough to resolve the oscillations of the transverse intensity distribution of the higher order transverse modes. Otherwise, this number can have the same size as the number of transverse grid points used for thermal FEA. The latter one also holds **Number of grid points in z-direction**.

Definition of the **Stretch factor in x- and y-direction related to beam diameter** strongly depends on N_{\max} and on the pump light distribution as described in the online help. For $N_{\max}=0$ it may be necessary to define a stretch factor equal to 2 to take into account the overlap between fundamental mode and absorbed pump power distribution.

2.2 Tab Rate Equations

Though some entries of this tab already can be defined in the window **Laser Power Output** of the LASCAD program, it is necessary to define them in this tab once more. For a comparison

with the results obtained for time independent laser power output, the entries must be defined identical.

2.3 Tab CW Operation

The default value of 10 ns for **Time resolution** turned out to be appropriate for common laser configurations. However, this entry, as well as the length of the **Time period used for simulation**, should be controlled by inspecting the results of the calculation.

2.4 Beam Quality

The beam quality factors M_x^2 and M_y^2 are computed according to Siegman and Townsend¹ using the expressions

$$M_x^2(t) = \sum_{i=1}^M (2p_i + 1)c_i(t) \quad (4)$$

and

$$M_y^2(t) = \sum_{i=1}^M (2q_i + 1)c_i(t) . \quad (5)$$

Here p_i and q_i are the transverse mode orders of the i -th gaussian mode in x - and y -direction, respectively. The coefficients $c_i(t)$ are defined by

$$c_i(t) = \frac{P_{i,out}(t)}{P_{C,out}(t)} , \quad (6)$$

where $P_{C,out}(t)$ is the total power output at time t , and $P_{i,out}(t)$ are the power outputs of the individual modes.

2.5 Results

After clicking **Calculate** in the DMA GUI, the window **DMA Calculation** opens, showing the progress of the computation. Among the information shown in this window, for instance, the value for **Power output for TEM00 mode using CW time independent recursion formula** can be used for comparison with the result obtained by the corresponding computation, which can be started by the use of the LASCAD window **Laser Power Output**.

The power output of individual modes averaged over the last quarter of simulation time is shown at the end of the calculation. The time average is restricted to the last quarter of simulation time, to reduce the influence of output spiking, as shown in Fig. 1. If spiking extends into this last quarter, it is recommended to increase the entry for the **Time period of simulation** in the tab **CW Operation**. In the following, if nothing else is specified, physical quantities are assumed to be averaged in the above way.

If, for the calculation, the cavity configuration according to Tutorial No. 1 is used together with the default settings for entries of the tabs of the DMA GUI except of $N_{max} = 3$, the results shown in Tab. 1 are obtained.

¹ A.E. Siegman and S. Townsend, "Output beam propagation and beam quality from a multimode stable-cavity laser", IEEE Journal Quantum Electron., **29**, 1212–1217 (1993)

Mode (0,3):	1.23341
Mode (0,2):	0.598921
Mode (0,1):	0.560508
Mode (1,2):	0.550806
Mode (3,0):	0.545464
Mode (1,3):	0.491664
Mode (0,0):	0.456887
Mode (3,1):	0.417439
Mode (1,1):	0.413295
Mode (1,0):	0.346157
Mode (3,3):	0.323292
Mode (2,2):	0.321322
Mode (2,1):	0.305183
Mode (2,0):	0.160791
Mode (3,2):	0.149285
Mode (2,3):	7.35643e-040

Tab. 1: Power output [W] of individual modes averaged over last quarter of simulation time obtained for the cavity configuration according to Tutorial No. 1

These results clearly show the influence of the strongly astigmatic distribution of the absorbed pump power density used in Tutorial No. 1, which has a gaussian distribution in the x-z-plane and a super-gaussian distribution with exponent 10 in the y-z-plane. Accordingly, the power of Modes (n,m) and (m,n) is not equal, for instance, Mode (0,3) has a power of 1.31 W, and the Mode (3,0) only 0.57 W.

Correspondingly, the obtained beam qualities $M_x^2 = 3.24$ and $M_y^2 = 4.22$ are different for x- and y-direction.

The total power output is 6.87 W.

After closing DMA and LASCAD, the numerical results also can be shown by opening the file output.txt in the corresponding DMA directory.

To visualize results, open the DMA Viewer with the button **Show Results** of the DMA GUI. The drop-down box in the lower part of the Viewer window allows for showing several important 2D and 3D plots.

An important plot is the power output over time, which is shown in figure 1 . The computation starts with population inversion density $N(x,y,z,t)=0$. Since this is different from equilibrium condition, a spiking behavior can be seen at the beginning, which attenuates with increasing time, and finally approaches a constant value. Dependent on the cavity configuration, it may be necessary to increase the **Time period used for simulation** to achieve convergence.

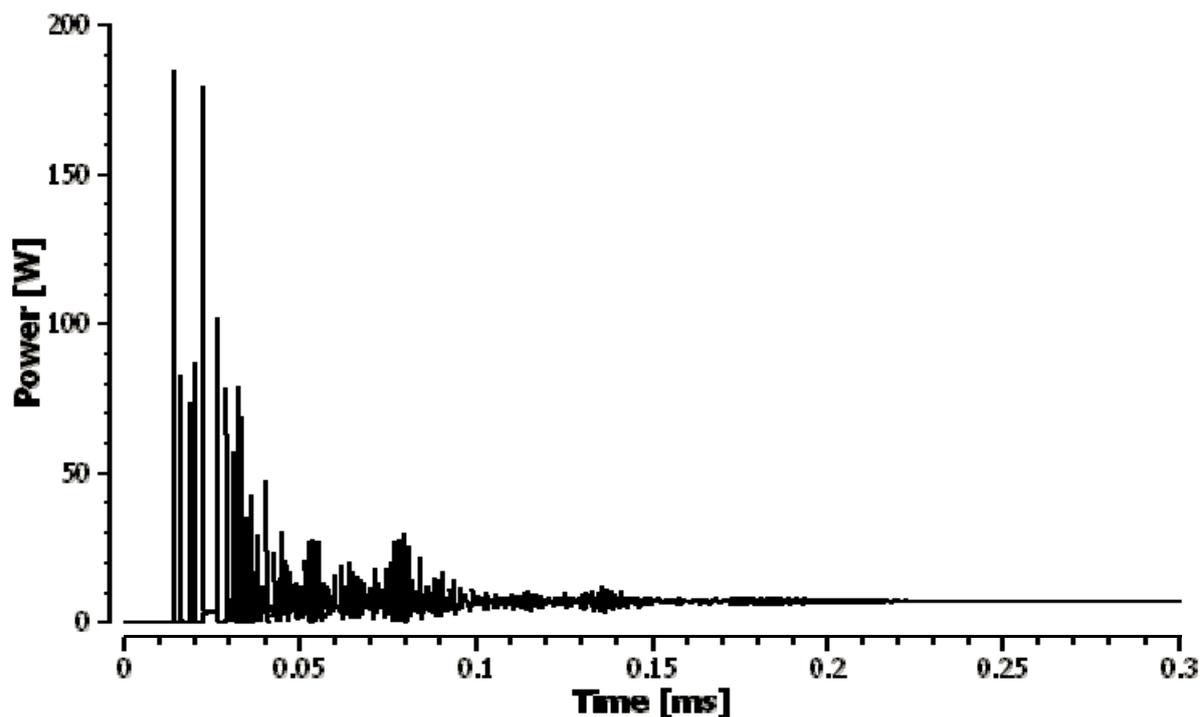


Fig. 1. Power output over time

Power output over time also can be shown for individual modes separately.

Also interesting are 2D plots of the beam quality over time and the spot of the Mode (0,0) along the cavity axis.

An interesting 3D plot is the **Beam profile in the crystal**, showing the superposition of the individual transverse modes, according to their contribution to the total power output. An example, once more based on the cavity configuration of Tutorial No.1, is shown in Fig. 2. This figure again shows the influence of the strongly astigmatic distribution of the absorbed pump power density used in Tutorial No. 1. The intensity distribution is strongly astigmatic. Transverse modes with higher order in y-direction are dominant.

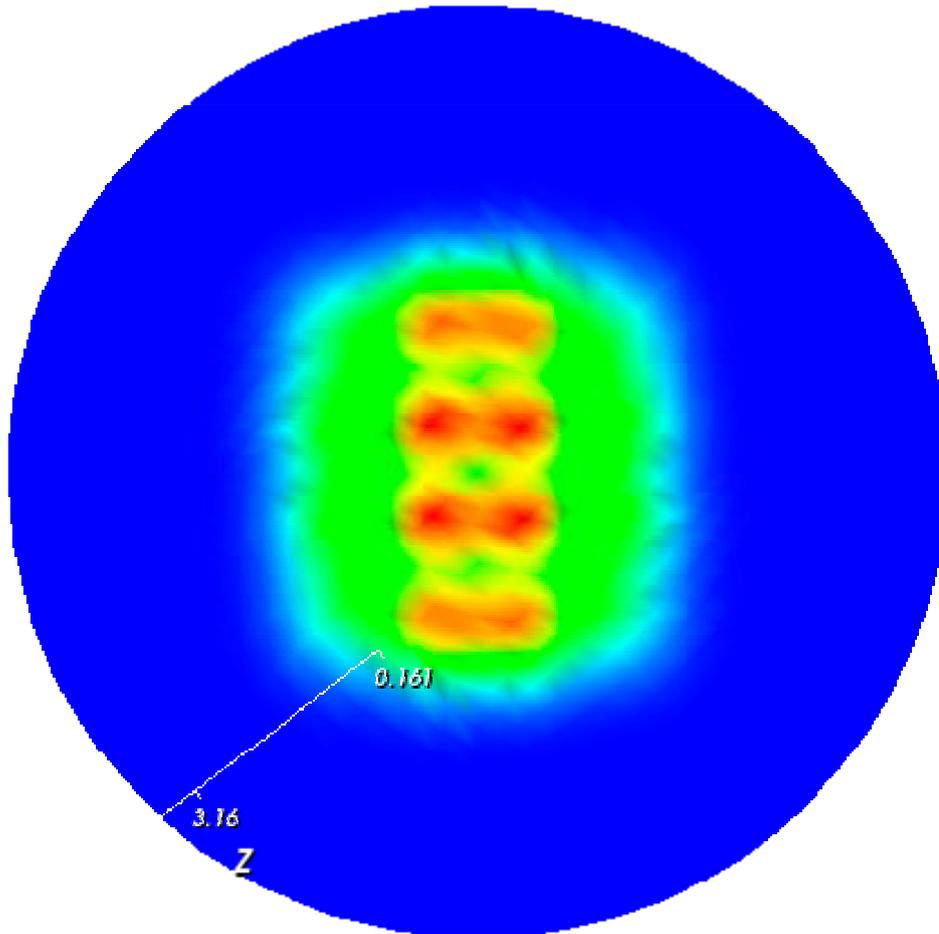


Fig. 2. Beam profile in the crystal

3 Modeling of Q-Switched Operation

Two different modes of pumping are available, cw pumping and pulsed pumping which can be selected from a drop-down box in the frame **Pumping** in the Tab Q-switch. Pulsed pumping can be used to create an approximate model for passive Q-switching, see section 3.3.

3.1 CW Pumping

For cw pumping a predefined number of subsequent pulses can be computed, which are triggered with constant repetition frequency.

During the load period onset of laser oscillation is suppressed by introducing a high artificial cavity loss in the rate equations, which can be defined in the box **Q-switch induced loss during load phase**. The default value 0.8 of this parameter is usually appropriate. Since no stimulated emission takes place during the load period, a high population inversion is generated.

If an opening period >0 is defined, the artificial Q-switch loss is not removed at once, but continuously reduced to the normal cavity loss during the defined opening period. However, this parameter only has a minor influence on pulse energy and shape.

Appropriate definition of the pulse period is very important. This quantity does not represent the physical pulse width, it only defines a time domain used for the computation of the pulse. Since population inversion and photon density change strongly during pulse development, it is necessary to define a large number of time steps during the pulse period to get short time steps. Since pulse build-up can be delayed dependent on the cavity configuration, it may be necessary to

make the pulse period much longer than the pulse width to prevent the pulse extending into the relaxation period.

Load period + opening period + pulse period must be smaller than the **pulse repetition period**. The remaining time is the relaxation period, representing a buffer zone between pulse period and new load period, if multiple pulses are computed. The number of time steps during the relaxation period can be small, since population inversion and photon density do not change much.

3.2 Results for CW Pumping

If the cavity configuration according to Tutorial No. 1 is used once more together with the default settings for entries of the tabs of the DMA GUI, except that only the fundamental mode is involved ($N_{\max}=0$), the calculation delivers the following result for the last pulse from a series of three:

```
Power output averaged over pulse repetition period [W] = 2.92419
Pulse energy [mJ] = 0.194946
Pulse width (FWHM) [ns] = 5.15
Average Beam Quality M^2 in x-direction = 1
Average Beam Quality M^2 in y-direction = 1
Peak power output 32216.1 [W] at time 0.000198894 [s]
```

The peak power output provides important information concerning damaging of optical components.

After zooming in the 2D plot **Power output over time**, as described in **DMA Viewer Help**, the shape of the pulses can be visualized. An example is displayed in Fig. 3, showing a typical pulse shape.

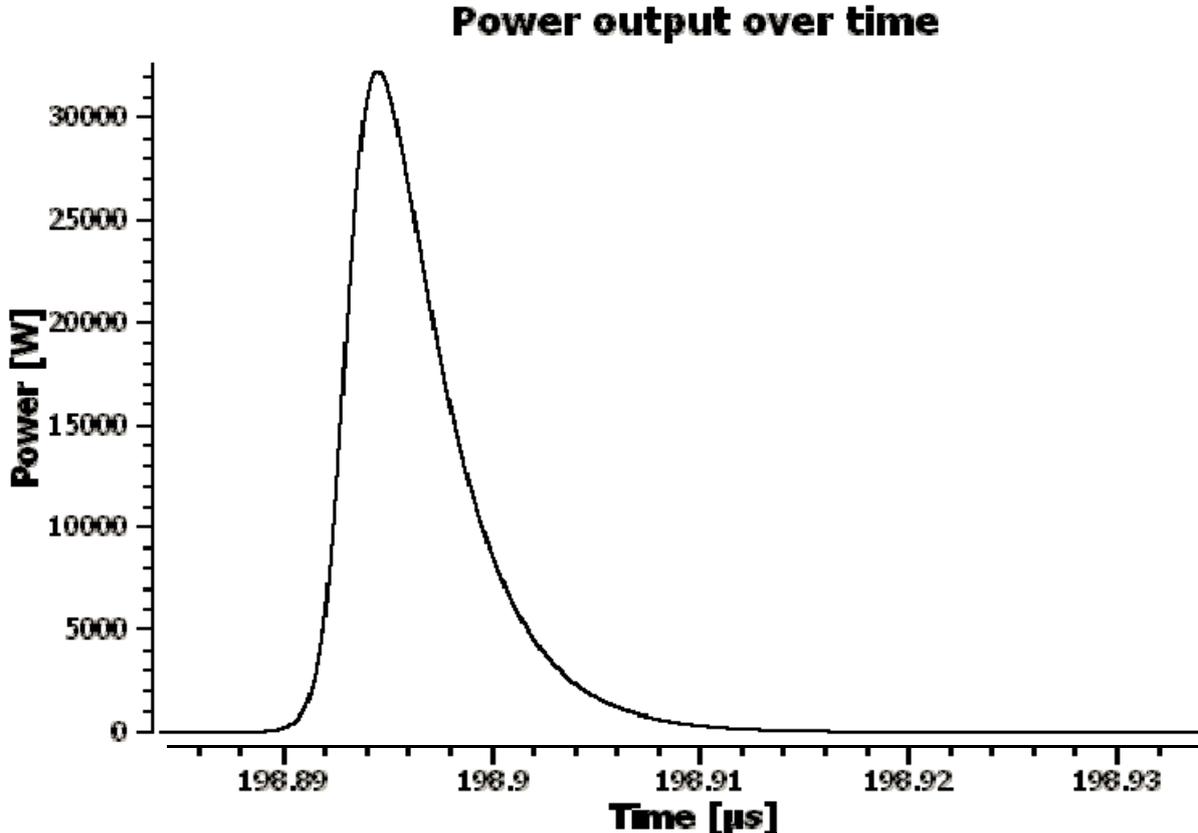


Fig. 3. Typical pulse shape for cw pumping

3.3 Pulsed Pumping

Alternatively to cw pumping, **Pulsed synchronous with Q-switch** can be selected as **Mode of pumping**. This input can be used to create an approximate model for passive Q-switching. In case of pulsed pumping it is assumed that pumping starts and ends synchronous with the load period. Since the frequency of pulsed pumping is typically much smaller than the pulse repetition frequency in case of cw pumping, the time-lag between two pulses is large. Therefore, we have assumed, that no interaction between subsequent pulses takes place, and have restricted the number of pulses to one. The time period used for computation ends with the pulse period.

Nevertheless the **Pulse repetition frequency** has to be defined by the following reason. The present version of LASCAD uses a time independent code for the thermal analysis. Therefore, in case of pulsed pumping, the time averaged pump power has to be used as input for the FEA. The accuracy of this approximation is sufficient, if the pump frequency is greater than 60 Hz. To obtain the real pump power during the pump pulse, LASCAD divides the value of the pump power defined in the FEA window by the product **Pulse repetition frequency x Load period**. Therefore both entries have to be defined in the tab **Q-switch**. For low pump frequencies, or single shot pumping, a time dependent thermal FEA is under development.

Since for pulsed pumping a higher population inversion can usually be achieved, it is preferable to use a smaller value for the reflectivity of the output mirror than for cw pumping. Using again the configuration according to Tutorial No. 1 with $N_{\max} = 0$ and **Reflectivity of output mirror** = 0.5 in the tab **Rate Equations**, together with the entries in tab Q-switch shown in Fig. 4, the following results are obtained:

```
Pulse 0: M^2_x = 1 M^2_y = 1
Power output averaged over pulse repetition period [W] = 71.8257
Pulse energy [mJ] = 14.3687
Pulse width (FWHM) [ns] = 0.495
Average Beam Quality M^2 in x-direction = 1
Average Beam Quality M^2 in y-direction = 1
Peak power output 2.3572e+007 [W] at time 0.000200002 [s]
Power averaged over whole simulation time [W]: 71.8257,
with whole simulation time [s]: 0.00020005.
Power output of individual modes averaged over computation pe-
riod [W]:
Mode (0,0): 71.8257
Power output summed up over all considered modes = 71.8257
```

Gaussian Modes	Rate Equations	CW Operation	Q-Switch	Apertures
Q-Switch				
Q-switch analysis			On	
Number of pulses			1	
Q-switch induced loss during load phase			0.8	
Pulse repetition frequency [kHz]			0.08	
Pulse repetition period [μ s]			12500	
Load period [μ s]			200	
Opening period (pulse build-up phase) [ns]			0	
Pulse period [ns]			50	
Relaxation period [μ s]			12300	
Number of time steps during load period			2000	
Number of time steps during opening period			1	
Number of time steps during pulse period			40000	
Number of time steps during relaxation period			1000	
Pumping				
Mode of pumping			Pulsed synchronous with Q-switch	

Fig. 4. Entries of tab Q-switch for pulsed pumping

Using the input values shown in Fig. 4 the pulse shape shown in Fig. 5 is obtained.

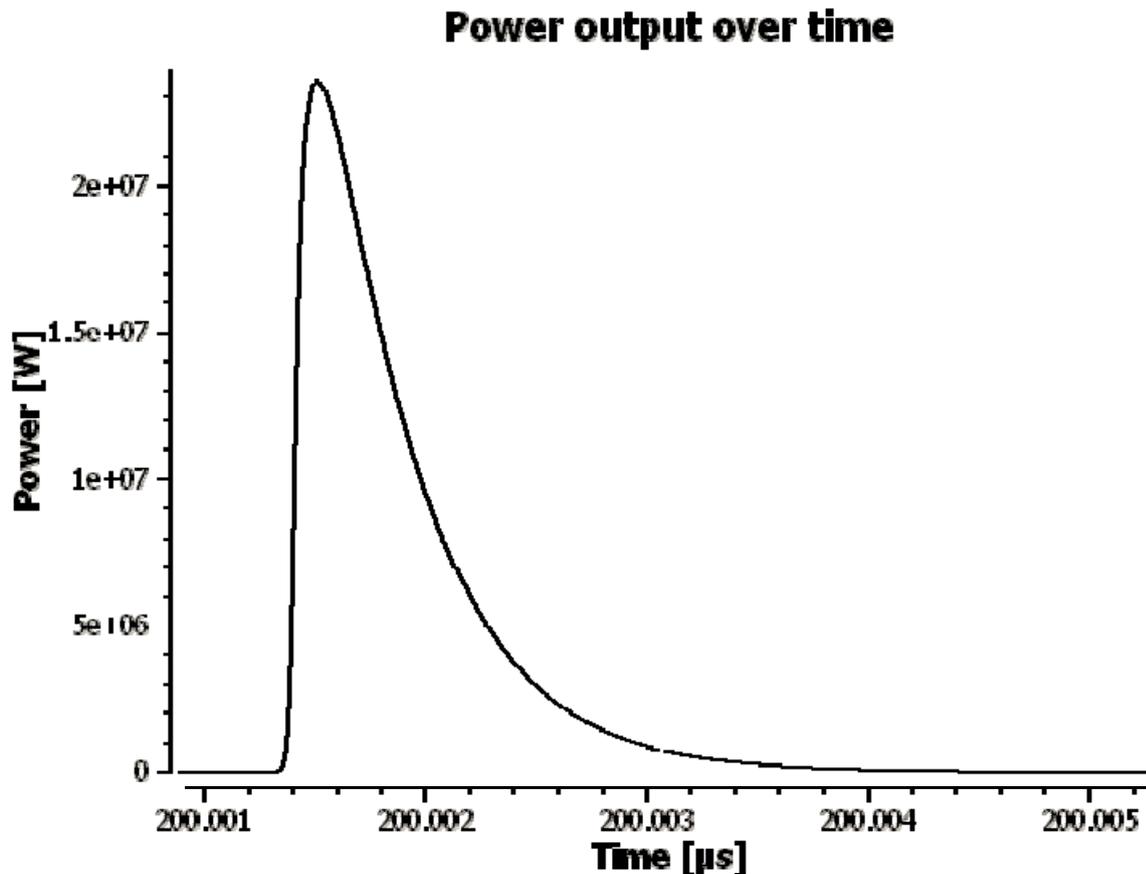


Fig. 5. Pulse shape obtained for pulsed pumping

4 Modeling the Effect of Apertures

The entries of the tab **Apertures** allow for modeling hard-edged and gaussian apertures, as well as hard edged, gaussian and super-gaussian mirrors.

4.1 Hard-Edged Apertures and Mirrors

The shape of hard-edged apertures or mirrors can be elliptical or rectangular. Select the corresponding item in the drop-down box **Type of Aperture**.

If, for the position of the aperture, a value < 0 is defined, the program assumes that the aperture is represented by the output mirror (right mirror). The radii of the aperture can be different in x- and y-direction.

Use again the configuration according to Tutorial No. 1 with $N_{\max} = 3$ as in Sect. 2.4 together with a circular end mirror with radius 0.2 mm, and run the calculation for cw operation. The obtained results shown in Tab. 2 indicate that, different from Tab. 1, due to the finite radius of the output mirror, only the modes (0,1), (1,0), and (0,0) contribute to the output power. This result is confirmed, if we are looking at the beam profile in the crystal shown in Fig. 6, and compare it with Fig. 2.

Correspondingly, the power output is reduced from 6.9 to 4 W, compared with Sect. 2.5 where no aperture is applied. However, the beam qualities are improved to $M_x^2 = 1.71$ and $M_y^2 = 1.92$.

Mode (0,1):	1.83459
Mode (1,0):	1.42093
Mode (0,0):	0.721171
Mode (1,1):	5.85802e-042
Mode (2,0):	4.07249e-042
Mode (0,2):	2.36935e-042
Mode (2,1):	9.62273e-043
Mode (1,2):	7.98643e-043
Mode (3,0):	6.69612e-043
Mode (0,3):	4.63281e-043
Mode (2,2):	3.62786e-043
Mode (3,1):	3.49377e-043
Mode (1,3):	2.9093e-043
Mode (3,2):	2.12117e-043
Mode (2,3):	2.08619e-043
Mode (3,3):	1.53504e-043

Tab. 2. Power output of individual modes averaged over last quarter of simulation time. For the computation the cavity configuration according to Tutorial No. 1 has been used, but with finite radius of the output mirror equal to 0.2 mm

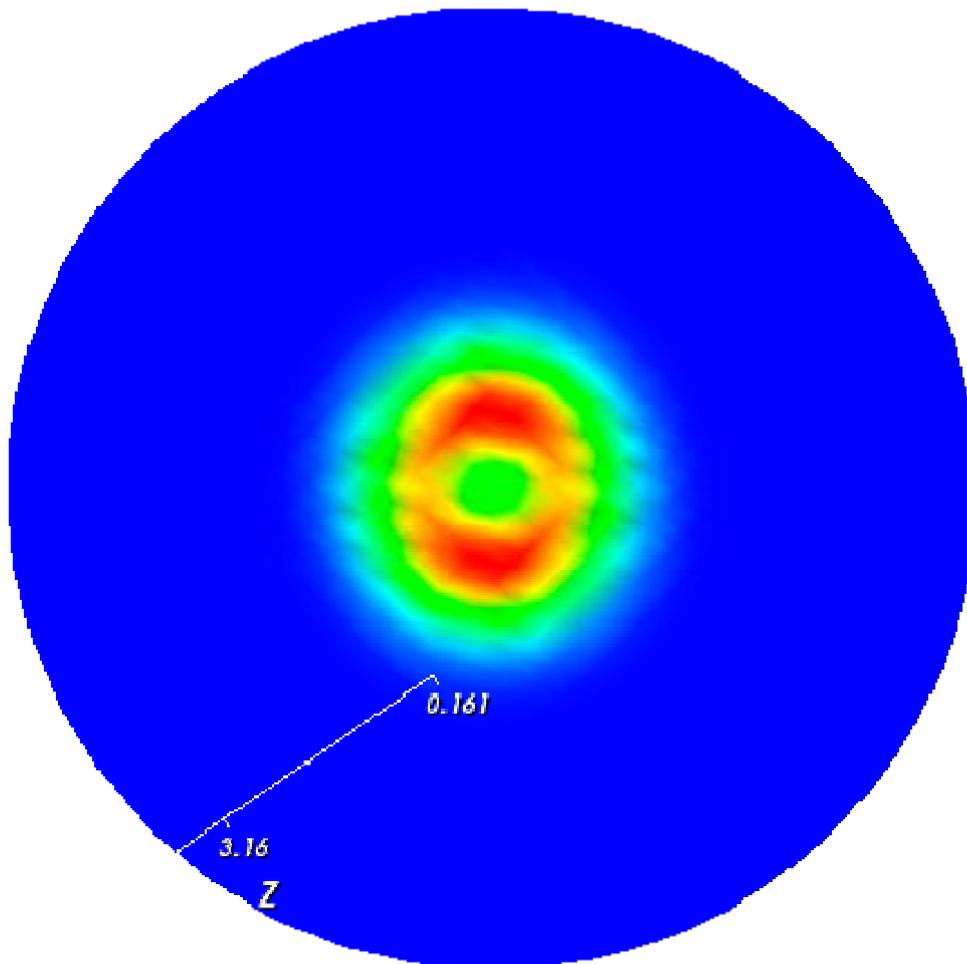


Fig. 6. Beam profile in the crystal for a cavity with output mirror of radius 0.2 mm

4.2 Super-gaussian Output Mirrors

The tab **Apertures** allows for defining gaussian and super-gaussian output mirrors. For this purpose, the option **Gaussian** has to be selected for **Type of aperture** and a value <0 has to be entered for the position of the aperture. In this case, the reflectivity of the output mirror is defined by

$$R(x, y) = R_0 \exp\left(-2\left|\frac{x}{w_{trx}}\right|^{SG} - 2\left|\frac{y}{w_{try}}\right|^{SG}\right) + R_{\min} \quad (4)$$

using a super-gaussian exponent $SG \geq 2$. R_{\min} is a peripheral bottom reflectivity. For R_0 the program uses the value $R_0 = R_{\text{out}} - R_{\min}$ where R_{out} has to be defined as **Reflectivity of output mirror** in the tab **Rate Equations**.

Since super-gaussian apertures cannot be taken into account in the ABCD matrix algorithm, the inner part of the super-gaussian distribution is fitted by a gaussian approximation dependent on SG , as shown in Fig. 7 for $SG = 4$. This approximation seems to be appropriate, since only modes with a radius $< w_{\text{trx}}(y)$ deliver a significant contribution to overall mode shape. The obtained gaussian fit is used in the ABCD matrix algorithm.

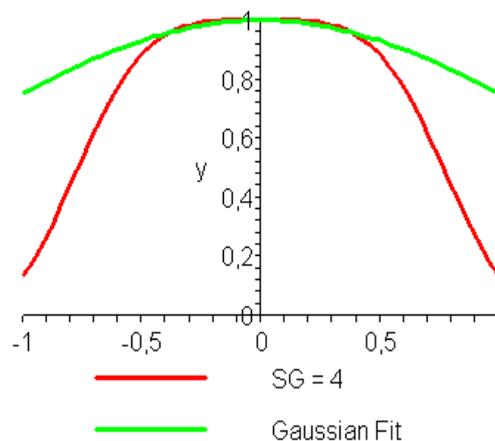


Fig. 7. Gaussian fit of a super-gaussian reflectivity distribution with $SG=4$

Once more, we use the configuration according to Tutorial No. 1 with $N_{\text{max}}=3$, as in Sect. 2.4 and 4.1, together with the inputs shown in Fig. 8. Running the calculation for cw operation, we obtain the results shown in Tab. 3. Now, as one can see, due to the super-gaussian output mirror, only the fundamental mode (0,0) is excited. Correspondingly, the beam profile displayed in Fig. 9 shows the well known shape of a gaussian fundamental mode. Surprisingly, the power of this mode is 4.22 W. This is relatively high compared with the calculation presented in Sect. 2.4, which delivered 6.87 W for the total power of all modes.

Compared with the result for a hard-edged aperture the power output increases from 4 to 4.22 W for a supergaussian aperture, whereas the beam quality improves considerably to 1 for x and y-direction.

Gaussian Modes	Rate Equations	CW Operation	Q-Switch	Apertures
Resonator contains aperture				True
Aperture type				Gaussian
Position of aperture along z-axis (<0 for pos. at right mirror) [mm]				-1
1/e ² -radius of gaussian transmission or reflection profile, x-dir. [mm]				0.2
1/e ² -radius of gaussian transmission or reflection profile, y-dir. [mm]				0.2
Peripheral bottom reflectivity in case of (super-) gaussian mirrors.				0
Super gaussian index of outcoupling mirror				4

Fig. 8. Entries of tab Q-switch Apertures for an output mirror with super-gaussian reflectivity

```

Mode (0,0): 4.22301
Mode (0,1): 3.01251e-041
Mode (1,0): 1.1211e-041
Mode (0,2): 8.25512e-042
Mode (1,1): 6.72075e-042
Mode (0,3): 4.86947e-042
Mode (1,2): 4.73229e-042
Mode (2,0): 4.64064e-042
Mode (2,1): 4.03505e-042
Mode (1,3): 3.73545e-042
Mode (2,2): 3.60176e-042
Mode (2,3): 3.24972e-042
Mode (3,0): 3.24888e-042
Mode (3,1): 3.21808e-042
Mode (3,2): 3.16512e-042
Mode (3,3): 3.05088e-042

```

Tab. 3. Power output [W] of individual modes averaged over last quarter of simulation time. For the computation the cavity configuration according to Tutorial No. 1 has been used, but with a super-gaussian output mirror, according to the entries shown in Fig. 8 .

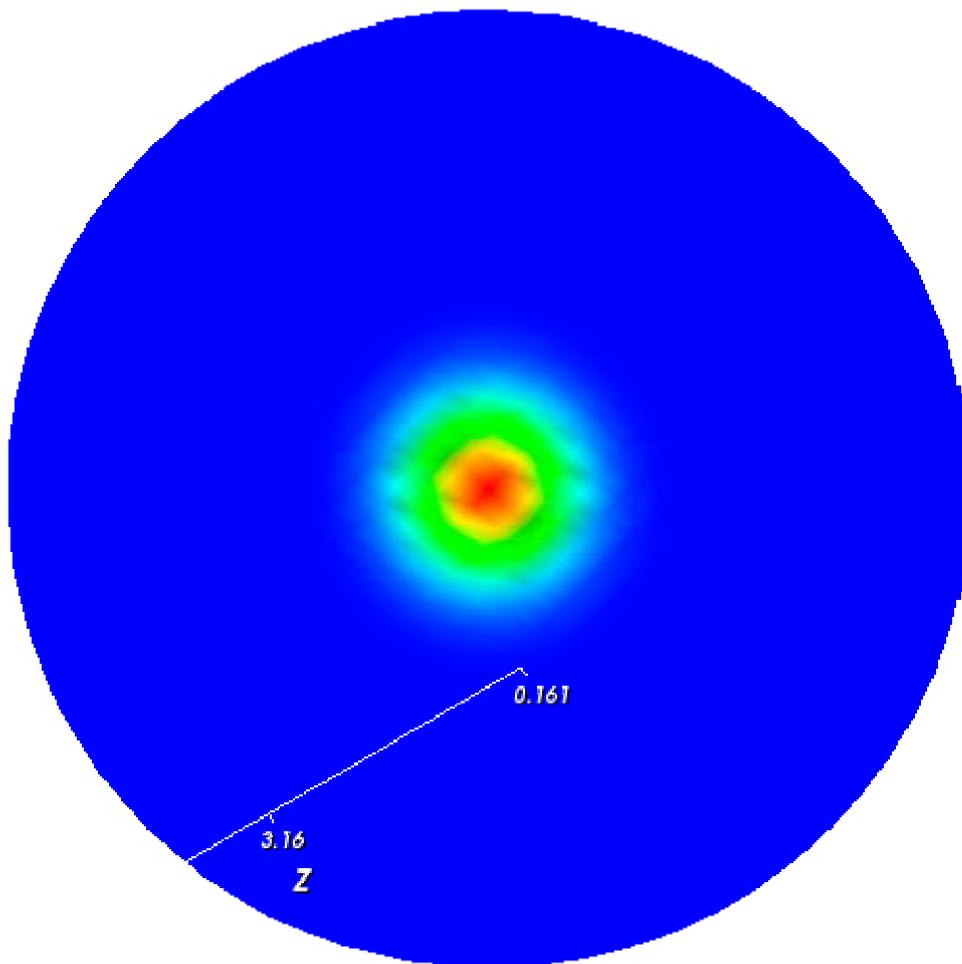


Fig. 9. Beam profile in the crystal for a cavity with super-gaussian output mirror

5 Conclusions

The results presented in the above sections demonstrate, that the DMA code for dynamic analysis of multimode competition and Q-switch operation is a very versatile tool to compute the time dependent interaction of a set of individual transverse modes, beam quality, laser power output, intensity distribution of the transverse mode structure, pulse shape for cw and pulsed pumping, peak intensity of pulses etc.. In this way it provides the laser engineer with important information to optimize a laser cavity configuration.